

Triangles

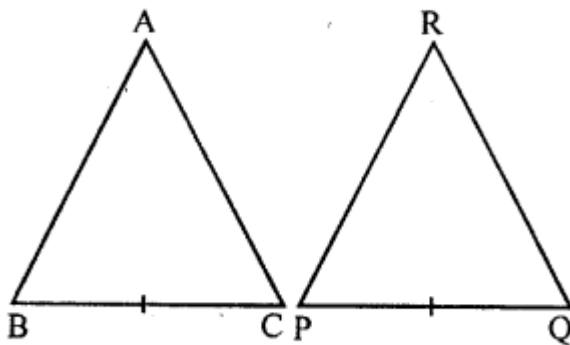
Question 1.

It is given that $\triangle ABC \cong \triangle RPQ$. Is it true to say that $BC = QR$? Why?

Solution:

$$\triangle ABC \cong \triangle RPQ$$

\therefore Their corresponding sides and angles are equal



$$\therefore BC = PQ$$

\therefore It is not true to say that $BC = QR$

Question 2.

“If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the two triangles must be congruent.” Is the statement true? Why?

Solution:

No, it is not true statement as the angles should be included angle of there two given sides.

Question 3.

In the given figure, $AB=AC$ and $AP=AQ$. Prove that

(i) $\triangle APC \cong \triangle AQB$

(ii) $CP = BQ$

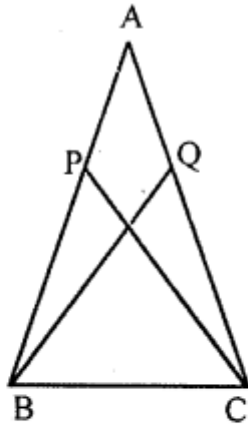
(iii) $\angle APC = \angle AQB$.

Solution:

Given : In the figure, $AB = AC$, $AP = AQ$

To prove :

- (i) $\triangle APC \cong \triangle AQB$ (ii) $CP = BQ$
(iii) $\angle APC = \angle AQB$



Proof : In $\triangle APC$ and $\triangle AQB$

$AC = AB$ (Given)

$AP = AQ$ (Given)

$\angle A = \angle A$ (Common)

(i) $\therefore \triangle APC \cong \triangle AQB$ (SAS axiom)

(ii) $BQ = CP$ (c.p.c.t.)

(iii) $\angle APC = \angle AQB$ (c.p.c.t.)

Question 4.

In the given figure, $AB = AC$, P and Q are points on BA and CA respectively such that $AP = AQ$. Prove that

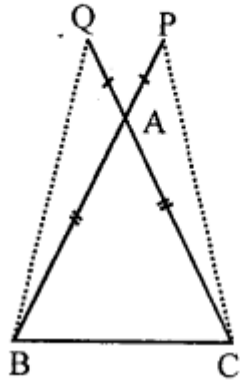
(i) $\triangle APC \cong \triangle AQB$

(ii) $CP = BQ$

(iii) $\angle ACP = \angle ABQ$.

Solution:

Given : In the given figure, $AB = AC$
 P and Q are point on BA and CA produced
 respectively such that $AP = AQ$



- To prove :** (i) $\triangle APC \cong \triangle AQB$
 (ii) $CP = BQ$
 (iii) $\angle ACP = \angle ABQ$

Proof : In $\triangle APC$ and $\triangle AQB$

$AC = AB$ (Given)

$AP = AQ$ (Given)

$\angle PAC = \angle QAB$ (Vertically opposite angle)

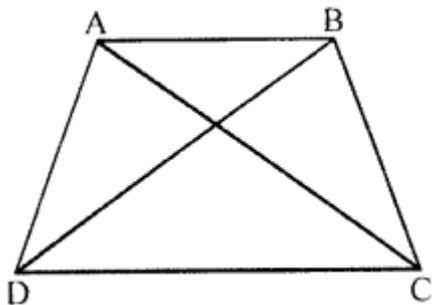
(i) $\therefore \triangle APC \cong \triangle AQB$ (SAS axiom)

$\therefore CP = BQ$ (c.p.c.t.)

$\angle ACP = \angle ABQ$ (c.p.c.t.)

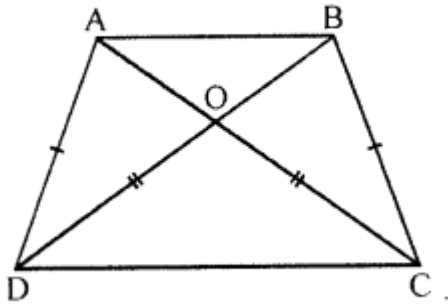
Question 5.

In the given figure, $AD = BC$ and $BD = AC$. Prove that :
 $\angle ADB = \angle BCA$ and $\angle DAB = \angle CBA$.



Solution:

Given : In the figure, $AD = BC$, $BD = AC$



To prove :

- (i) $\angle ADB = \angle BCA$
- (ii) $\angle DAB = \angle CBA$

Proof : In $\triangle ADB$ and $\triangle ACB$

$$AB = AB \quad (\text{common})$$

$$AD = BC \quad (\text{given})$$

$$DB = AC \quad (\text{given})$$

$$\therefore \triangle ADB \cong \triangle ACB \quad (\text{SSS axiom})$$

$$\therefore \angle ADB = \angle BCA \quad (\text{c.p.c.t.})$$

$$\angle DAB = \angle CBA \quad (\text{c.p.c.t.})$$

Question 6.

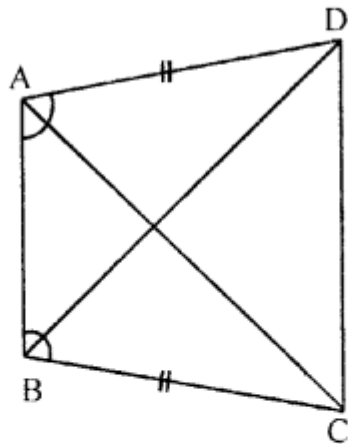
In the given figure, ABCD is a quadrilateral in which $AD = BC$ and $\angle DAB = \angle CBA$.

Prove that

- (i) $\triangle ABD \cong \triangle BAC$
- (ii) $BD = AC$
- (iii) $\angle ABD = \angle BAC$.

Solution:

Given : In the figure ABCD is a quadrilateral
in which $AD = BC$
 $\angle DAB = \angle CBA$



To prove :

- (i) $\triangle ABD \cong \triangle BAC$ (ii) $BD = AC$
(iii) $\angle ABD = \angle BAC$

Proof : In $\triangle ABD$ and $\triangle BAC$

$AB = AB$ (Common)

$\angle DAB = \angle CBA$ (Given)

$AD = BC$ (Given)

(i) $\therefore \triangle ABD \cong \triangle BAC$ (SAS axiom)

(ii) $\therefore BD = AC$ (c.p.c.t.)

(iii) $\angle ABD = \angle BAC$ (c.p.c.t.)

Question 7.

In the given figure, $AB = DC$ and $AB \parallel DC$. Prove that $AD = BC$.

Solution:

Given : In the given figure,

$AB = DC, AB \parallel DC$

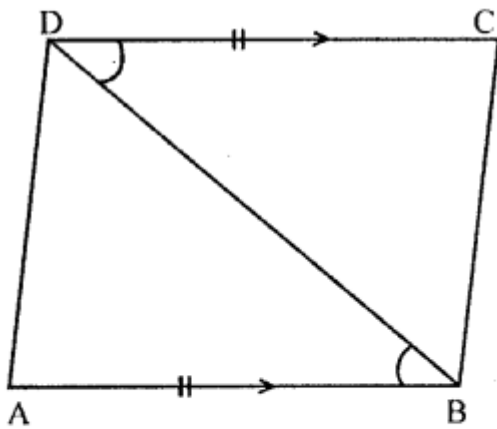
To prove : $AD = BC$

Proof : $\because AB \parallel DC$

$\therefore \angle ABD = \angle CDB$ (Alternate angles)

In $\triangle ABD$ and $\triangle CDB$

$AB = DC$ (Given)



$\angle ABD = \angle CDB$ (Alternate angles)

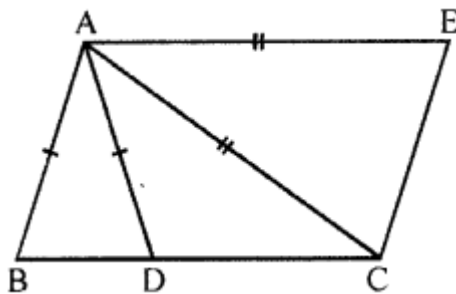
$BD = BD$ (Common)

$\therefore \triangle ABD \cong \triangle CDB$ (SAS axiom)

$\therefore AD = BC$ (c.p.c.t.)

Question 8.

In the given figure. $AC = AE, AB = AD$ and $\angle BAD = \angle CAE$. Show that $BC = DE$.



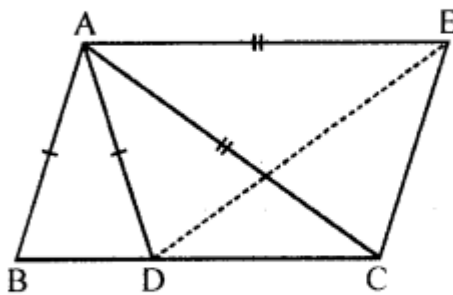
Solution:

Given : In the figure, $AC = AE$, $AB = AD$

$\angle BAD = \angle CAE$

To prove : $BC = DE$

Construction : Join DE .



Proof : In $\triangle ABC$ and $\triangle ADE$

$AB = AD$ (given)

$AC = AE$ (given)

$\angle BAD + \angle DAC = \angle DAC + \angle CAE$

$\Rightarrow \angle BAC = \angle DAE$

$\therefore \triangle ABC \cong \triangle ADE$ (SAS axiom)

$\therefore BC = DE$ (c.p.c.t.)

Question 9.

In the adjoining figure, $AB = CD$, $CE = BF$ and $\angle ACE = \angle DBF$. Prove that

(i) $\triangle ACE \cong \triangle DBF$

(ii) $AE = DF$.

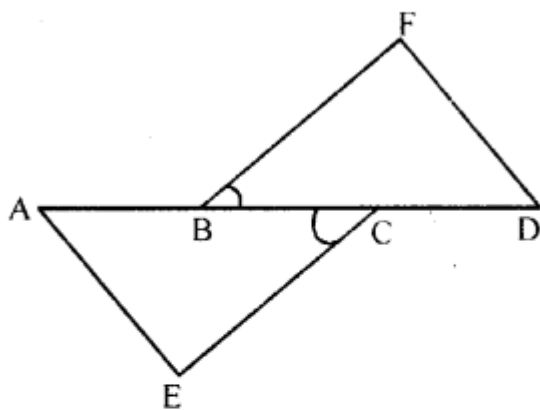
Solution:

Given : In the given figure,

$$AB = CD$$

$$CE = BF$$

$$\angle ACE = \angle DBF$$



To prove : (i) $\triangle ACE \cong \triangle DBF$

(ii) $AE = DF$

Proof : $\because AB = CD$

Adding BC to both sides

$$AB + BC = BC + CD$$

$$\Rightarrow AC = BD$$

Now in $\triangle ACE$ and $\triangle DBF$

$$AC = BD \quad \text{(Proved)}$$

$$CE = BF \quad \text{(Given)}$$

$$\angle ACE = \angle DBF \quad \text{(Given)}$$

$$(i) \therefore \triangle ACE \cong \triangle DBF \quad \text{(SAS axiom)}$$

$$\therefore AE = DE \quad \text{(c.p.c.t.)}$$

Question 10.

In the given figure, $AB = AC$ and D is mid-point of BC. Use SSS rule of congruency to show that

(i) $\triangle ABD \cong \triangle ACD$

(ii) AD is bisector of $\angle A$

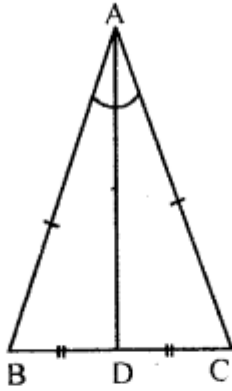
(iii) AD is perpendicular to BC.

Solution:

Given : In the given figure, $AB = AC$

D is mid point of BC

$\therefore BD = DC$



To prove :

(i) $\triangle ABD \cong \triangle ACD$

(ii) AD is bisector of $\angle A$

(iii) $AD \perp BC$

Proof : In $\triangle ABD$ and $\triangle ACD$

$AB = AC$ (Given)

$BD = DC$ (Given)

$AD = AD$ (Common)

(i) $\therefore \triangle ABD \cong \triangle ACD$

(ii) $\angle BAD = \angle CAD$ (c.p.c.t.)

\therefore AD is the bisector of $\angle A$

(iii) $\angle ADB = \angle ADC$

But $\angle ADB + \angle ADC = 180^\circ$ (Linear pair)

$\therefore \angle ADB = \angle ADC = 90^\circ$

$\therefore AD \perp BC$

Question 11.

Two line segments AB and CD bisect each other at O. Prove that :

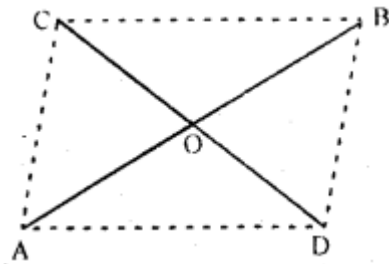
(i) $AC = BD$

(ii) $\angle CAB = \angle ABD$

(iii) $AD \parallel CB$

(iv) $AD = CB$.

Solution:



In $\triangle AOC$ and $\triangle BOD$

$OC = OD$ [AB and CD bisect each other]
and $AO = OB$

Also, $\angle AOC = \angle BOD$ (vertical opposite angles)

$\therefore \triangle AOC \cong \triangle BOD$

(By S.A.S. axiom of congruency)

(i) Then $AC = BD$ (c.p.c.t.)

(ii) Also $\angle CAO = \angle DBO$ (c.p.c.t.)

i.e. $\angle CAB = \angle ABD$

[$\because \angle CAO = \angle CAB$ and $\angle DBO = \angle ABD$]

(iii) We have in (ii) part

$\angle CAB = \angle ABD$

But these are Alternate angles

Hence, $AD \parallel CB$

(iv) In $\triangle AOD$ and $\triangle BOC$

$OC = OD$ and $BO = AO$

(AB and CD bisect each other)

and $\angle BOC = \angle AOD$

(vertical opposite angles)

$\therefore \triangle AOD \cong \triangle BOC$

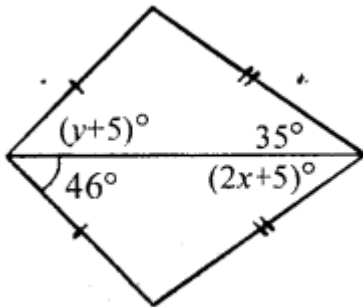
[By S.A.S. axiom of congruency]

Then, $AD = BC$ (c.p.c.t.)

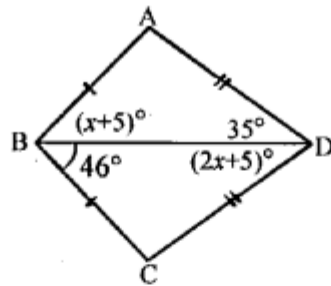
or $AD = CB$ (Q.E.D.)

Question 12.

In each of the following diagrams, find the values of x and y .



Solution:



In $\triangle ABD$ and $\triangle BCD$,

$$AB = BC \quad (\text{given})$$

$$AD = CD \quad (\text{given})$$

$$BD = BD \quad (\text{common})$$

$$\therefore \triangle ABD \cong \triangle BCD$$

(By S.S.S axiom of congruency)

$$\therefore \angle ABD = \angle CBD \quad (\text{c.p.c.t.})$$

$$\Rightarrow y + 5 = 46 \Rightarrow y = 46 - 5 \Rightarrow y = 41$$

$$\text{Also } \angle ADB = \angle BDC \quad (\text{c.p.c.t.})$$

$$\Rightarrow 35^\circ = (2x + 5)^\circ \Rightarrow 35 = 2x + 5$$

$$\Rightarrow 2x + 5 = 35 \Rightarrow 2x = 35 - 5 \Rightarrow 2x = 30$$

$$\Rightarrow x = \frac{30}{2} \Rightarrow x = 15$$

Exercise 10.2

Question 1.

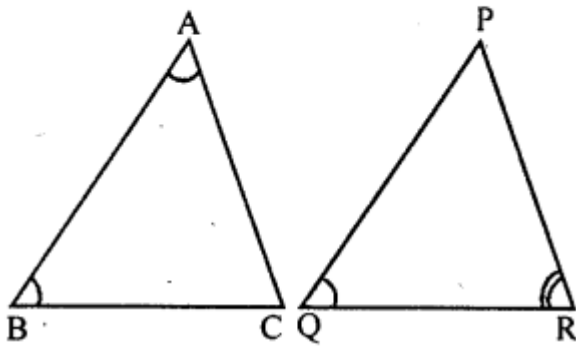
In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of APQR should be equal to side AB of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In $\triangle ABC$ and $\triangle PQR$

$$\angle A = \angle Q$$

$$\angle B = \angle R$$



$$AB = QP$$

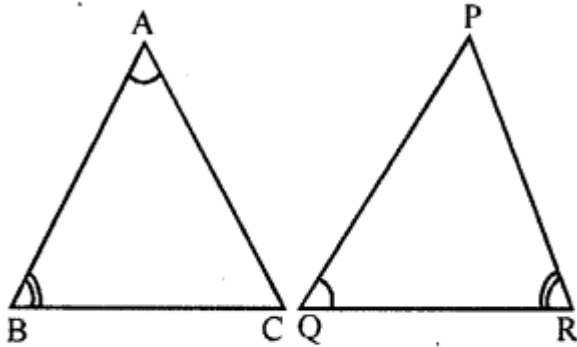
\therefore Two Δ s are congruent if their corresponding two angles and included sides are equal

Question 2.

In triangles ABC and PQR, $\angle A = \angle Q$ and $\angle B = \angle R$. Which side of APQR should be equal to side BC of AABC so that the two triangles are congruent? Give reason for your answer.

Solution:

In ΔABC and ΔPQR



$$\angle A = \angle Q$$

$$\angle B = \angle R$$

and their included sides AB and QR will be equal for their congruency

$$\therefore BC = PR \quad (\text{c.p.c.t.})$$

Question 3.

“If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles must be congruent”. Is the statement true? Why?

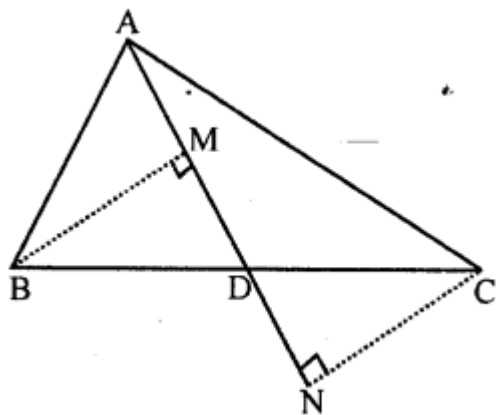
Solution:

The given statement can be true only if the corresponding (included) sides are equal otherwise not.

Question 4.

In the given figure, AD is median of $\triangle ABC$, BM and CN are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that $BM = CN$.

Solution:



Given : In $\triangle ABC$, AD is median BM and CN are perpendicular to AD from B and C respectively

To prove : $BM = CN$

Proof : In $\triangle BMD$ and $\triangle CND$

$BD = CD$ (AD is median)

$\angle M = \angle N$ (each 90°)

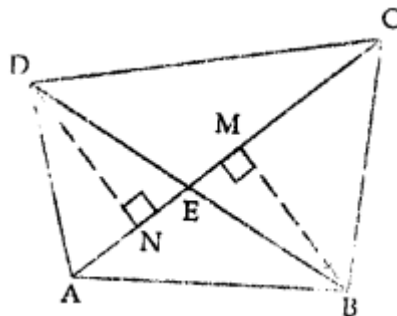
$\angle BDM = \angle CDN$
(Vertically opposite angles)

$\therefore \triangle BMD \cong \triangle CND$ (AAS axiom)

$\therefore BM = CN$ (c.p.c.t.)

Question 5.

In the given figure, BM and DN are perpendiculars to the line segment AC. If $BM = DN$, prove that AC bisects BD.



Solution:

Given : In the figure, BM and DN are perpendicular to AC

$$BM = DN$$

To prove : AC bisects BD *i.e.*, BE = ED

Construction : Join BD which intersects AC at E

Proof : In $\triangle BEM$ and $\triangle DEN$

$$BM = DN \quad (\text{Given})$$

$$\angle M = \angle N \quad (\text{each } 90^\circ)$$

$$\angle DEN = \angle BEM$$

(Vertically opposite angles)

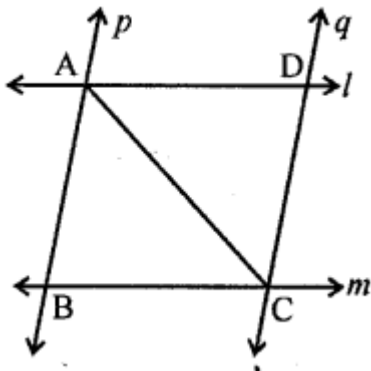
$$\therefore \triangle BEM \cong \triangle DEN \quad (\text{AAS axiom})$$

$$\therefore BE = ED$$

\Rightarrow AC bisects BD

Question 6.

In the given figure, l and m are two parallel lines intersected by another pair of parallel lines p and q. Show that $\triangle ABC \cong \triangle CDA$.



Solution:

In the given figure, two lines l and m are parallel to each other and lines p and q are also a pair of parallel lines intersecting each other at A, B, C and D. AC is joined.

To prove : $\triangle ABC \cong \triangle CDA$

Proof : In $\triangle ABC$ and $\triangle CDA$

$$AC = AC \quad (\text{Common})$$

$$\angle ACB = \angle CAD \quad (\text{Alternate angles})$$

$$\angle BAC = \angle ACD \quad (\text{Alternate angles})$$

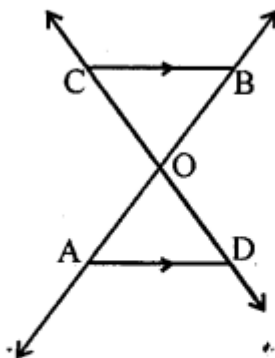
$$\therefore \triangle ABC \cong \triangle CDA \quad (\text{ASA axiom})$$

Question 7.

In the given figure, two lines AB and CD intersect each other at the point O such that $BC \parallel DA$ and $BC = DA$. Show that O is the mid-point of both the line segments AB and CD.

Solution:

Given : In the given figure, lines AB and CD intersect each other at O such that $BC \parallel AD$ and $BC = DA$



To prove : O is the mid point of AB and CD

Proof : $\triangle AOD$ and $\triangle BOC$

$$AD = BC \quad (\text{Given})$$

$$\angle OAD = \angle OBC \quad (\text{Alternate angles})$$

$$\angle ODA = \angle OCB \quad (\text{Alternate angles})$$

$$\therefore \triangle AOD \cong \triangle BOC \quad (\text{SAS axiom})$$

$$\therefore OA = OB \text{ and } OD = OC$$

$$\therefore O \text{ is the mid-point of AB and CD}$$

Question 8.

In the given figure, $\angle BCD = \angle ADC$ and $\angle BCA = \angle ADB$. Show that

(i) $\triangle ACD \cong \triangle BDC$

(ii) $BC = AD$

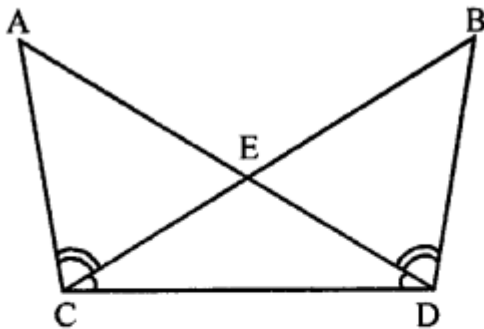
(iii) $\angle A = \angle B$.

Solution:

Given : In the given figure,

$$\angle BCD = \angle ADC$$

$$\angle BCA = \angle ADB$$



To prove :

(i) $\triangle ACD \cong \triangle BDC$ (ii) $BC = AD$

(iii) $\angle A = \angle B$

Proof : $\because \angle BCA = \angle ADB$

and $\angle BCD = \angle ADC$

Adding we get,

$$\angle BCA + \angle BCD = \angle ADB + \angle ADC$$

$$\Rightarrow \angle ACD = \angle BDC$$

Now in $\triangle ACD$ and $\triangle BDC$

$$CD = CD \quad \text{(Common)}$$

$$\angle ACD = \angle BDC \quad \text{(Proved)}$$

$$\angle ADC = \angle BCD \quad \text{(Given)}$$

(i) $\therefore \triangle ACD \cong \triangle BDC$ (ASA axiom)

$\therefore AD = BC$ (c.p.c.t.)

$\angle A = \angle B$ (c.p.c.t.)

Question 9.

In the given figure, $\angle ABC = \angle ACB$, D and E are points on the sides AC and AB respectively such that $BE = CD$. Prove that

(i) $\triangle EBC \cong \triangle DCB$

(ii) $\triangle OEB \cong \triangle ODC$

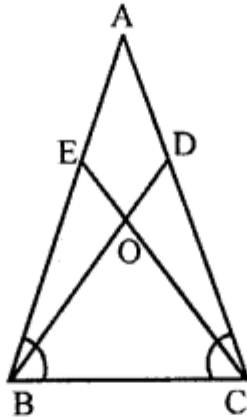
(iii) $OB = OC$.

Solution:

Given : In the given figure,

$$\angle ABC = \angle ACB$$

D and E are the points on AC and AB such



To prove : (i) $\triangle EBC \cong \triangle DCB$

(ii) $\triangle OEB \cong \triangle ODC$

(iii) $OB = OC$

Proof : In $\triangle ABC$,

$$\because \angle ABC = \angle ACB$$

$$\therefore AC = AB \quad (\text{Sides opposite to equal angles})$$

In $\triangle EBC$ and $\triangle DCB$,

$$EB = DC \quad (\text{Given})$$

$$BC = BC \quad (\text{Common})$$

$$\angle CBD = \angle DCB \quad (\because \angle ABC = \angle ACB)$$

(i) $\therefore \triangle EBC \cong \triangle DCB$ (SAS axiom)

$$\angle ECB = \angle DBC \quad (\text{c.p.c.t.})$$

Now in $\triangle OEB$ and $\triangle ODC$

$$BE = CD \quad (\text{Given})$$

$$\angle EBO = \angle DCO$$

$$\{\because \angle ABC - \angle DBC = \angle ACB - \angle OCB\}$$

$$\angle EOB = \angle DOC$$

(ii) $\therefore \triangle OEB \cong \triangle ODC$ (AAS axiom)

(iii) $\therefore OB = OC$ (c.p.c.t.)

Question 10.

ABC is an isosceles triangle with $AB=AC$. Draw $AP \perp BC$ to show that $\angle B = \angle C$.

Solution:

Given : $\triangle ABC$ is an isosceles triangle with

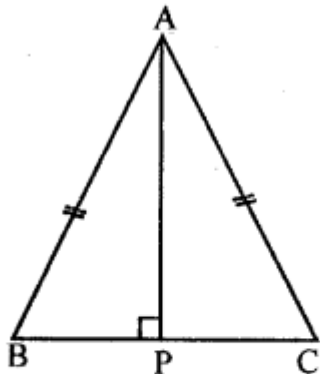
$AB = AC$

$AP \perp BC$

To prove : $\angle B = \angle C$

Proof : In right $\triangle APB$ and $\triangle APC$

Side $AP = AP$ (Common)



Hyp. $AB = AC$ (Given)

$\therefore \triangle APB \cong \triangle APC$ (RHS axiom)

$\therefore \angle B = \angle C$ (c.p.c.t.)

Question 11.

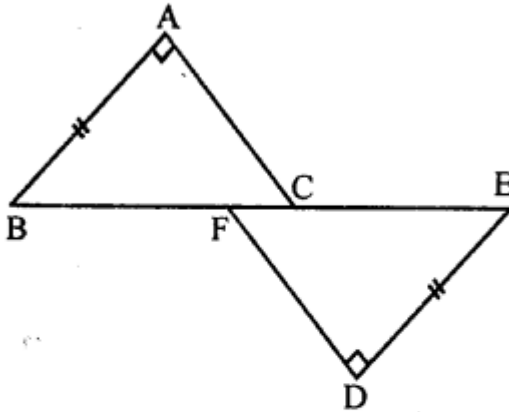
In the given figure, $BA \perp AC$, $DE \perp DF$ such that $BA = DE$ and $BF = EC$.

Solution:

Given : In the given figure,

$BA \perp AC, DE \perp DF$

$BA = DE, BF = EC$



To prove : $\triangle ABC \cong \triangle DEF$

Proof : $\because BF = CE$

Adding FC both sides

$$BF + FC = FC + CE$$

$$\Rightarrow BC = EF$$

Now in right $\triangle ABC$ and $\triangle DEF$

Side $AB = DE$ (Given)

Hyp. $BC = EF$ (Proved)

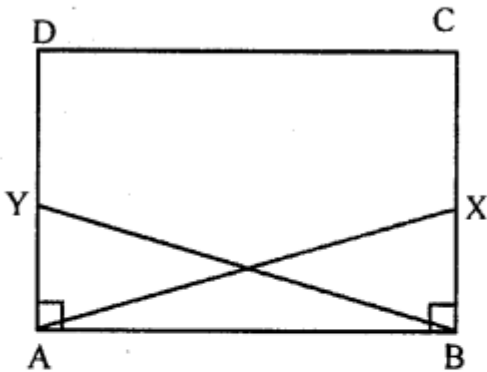
$\therefore \triangle ABC \cong \triangle DEF$

Question 12.

ABCD is a rectangle. X and Y are points on sides AD and BC respectively such that AY = BX. Prove that BY = AX and $\angle BAY = \angle ABX$.

Solution:

Given : In rectangle ABCD, X and Y are points on the sides AD and BC respectively such that AY = BX



To prove : $BY = AX$ and $\angle BAY = \angle ABX$

Proof : In $\triangle ABX$ and $\triangle ABY$

$AB = AB$ (Common)

$\angle A = \angle B$ (Each 90°)

$BX = AY$ (Given)

$\therefore \triangle ABX \cong \triangle ABY$ (SAS axiom)

$AX = BY$ (c.p.c.t.)

or $BY = AX$

and $\angle AXB = \angle BYA$ (c.p.c.t.)

Question 13.

(a) In the figure (1) given below, QX, RX are bisectors of angles PQR and PRQ respectively of $\triangle PQR$. If $XS \perp QR$ and $XT \perp PQ$, prove that

(i) $\triangle XTQ \cong \triangle XSQ$

(ii) PX bisects the angle P.

(b) In the figure (2) given below, $AB \parallel DC$ and $\angle C = \angle D$. Prove that

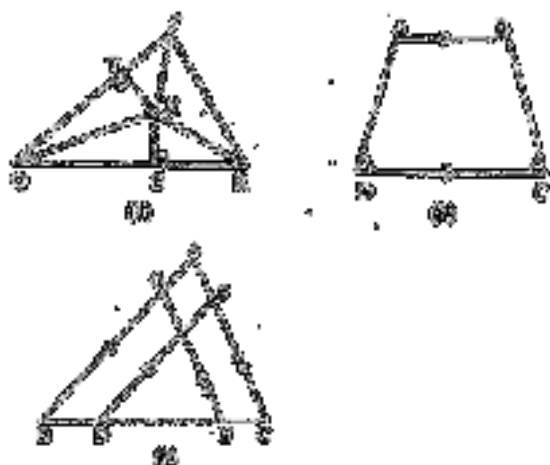
(i) $AD = BC$

(ii) $AC = BD$.

(c) In the figure (3) given below, $BA \parallel DF$ and $CA \parallel EG$ and $BD = EC$. Prove that,

(i) $BG = DF$

(ii) $EG = CF$.



Solution:

It is given that $AD \perp BC$, $BE \perp AC$ is the altitude of $\triangle ABC$ and $CF \perp AB$ is the altitude of $\triangle ABC$.
 To prove:
 (a) AD, BE, CF are concurrent.
 (b) P is the orthocenter of $\triangle ABC$.

To prove:

(a) AD, BE, CF are concurrent.

(b) P is the orthocenter of $\triangle ABC$.



Construction: Draw $AD \perp BC$.

Proof: In $\triangle ACF$ and $\triangle ABE$

$$\angle CAF = \angle BAE \quad (\text{Common})$$

$$\angle AFC = \angle AEB = 90^\circ \quad (\text{Given})$$

$$\angle ACF = \angle ABE \quad (CF \text{ is altitude of } \triangle ABC)$$

$$CF = BE \quad (\text{Common})$$

$$\therefore \triangle ACF \cong \triangle ABE \quad (\text{By A.A.S. rule of congruence})$$

$$AF = BE \quad (\text{Corresponding parts}) \quad \dots (1)$$

Now in $\triangle ACF$ and $\triangle ABE$

$$\angle CAF = \angle BAE \quad (\text{Common})$$

$$\angle AFC = \angle AEB = 90^\circ \quad (\text{Given})$$

$$\angle ACF = \angle ABE \quad (\text{By A.A.S. rule of congruence})$$

$$AF = BE \quad (\text{Common})$$

$$\therefore \triangle ACF \cong \triangle ABE \quad (\text{By A.A.S. rule of congruence})$$

$$\therefore AF = BE \quad (\text{Corresponding parts}) \quad \dots (2)$$

From (1) and (2), we get

$$AF = BE \quad \dots (3)$$

In $\triangle ACF$ and $\triangle ABE$

$$\angle ACF = \angle ABE = 90^\circ$$

$$\left[\begin{array}{l} \angle XTP = 90^\circ \text{ (given)} \\ \angle XTP = 90^\circ \text{ (construction)} \end{array} \right]$$

(hyp) $XP =$ (hyp) XP (common)
 $XT = XZ$ [From (3)]

$\therefore \Delta XTP \cong \Delta XZP$
 [By R.H.S. axiom of congruency

$\therefore \angle XPT = \angle XPZ$ (c.p.c.t.)

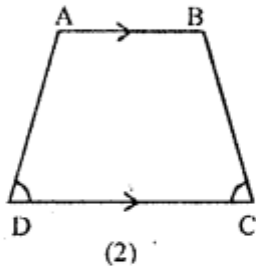
\therefore PX bisects the angle P. (Q.E.D.)

(b) In following figure

Given. $AB \parallel DC$ and $\angle C = \angle D$

To prove. (i) $AD = BC$

(ii) $AC = BD$



Construction. Draw $AE \perp CD$, $BF \perp CD$ and join A to C and B to D.

Proof. (i) In ΔAED and ΔBCF

$\angle AED = \angle BFC$ (each 90°)

[By construction $AE \perp CD$ and $BF \perp CD$]

$\angle D = \angle C$ (given)

$AE = BF$

[Distance between parallel lines are same]

$\therefore \Delta AED \cong \Delta BCF$

(By A.A.S. axiom of congruency

$AD = BC$ (c.p.c.t.) (1)

(ii) In ΔACD and ΔBCD

$\angle D = \angle C$ (Given)

$DC = DC$ (Common)

$AD = BC$ [From (1)]

$\therefore \Delta ACD \cong \Delta BCD$

(By S.A.S. axiom of congruency

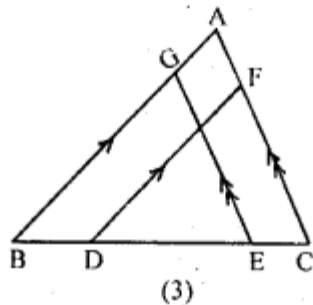
$\therefore AC = BD$ (c.p.c.t.) (Q.E.D.)

(c) In following figure

Given. $BA \parallel DF$ and $CA \parallel EG$ and $BD = EC$

To prove. (i) $BG = DF$

(ii) $EG = CF$



Proof. (i) In $\triangle BEG$ and $\triangle DCF$

$$\angle B = \angle D$$

($\because BA \parallel DF$, corresponding angles equal)

$$\angle E = \angle C$$

($\because CA \parallel EG$ corresponding angles equal)

and $BE = BC - EC = BC - BD = DC$

i.e. $BE = DC$

$\therefore \triangle BEG \cong \triangle DCF$

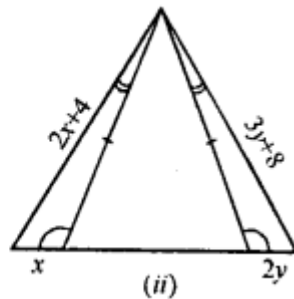
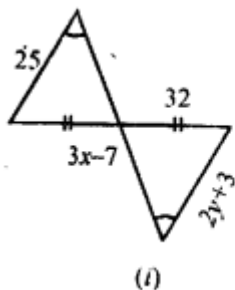
(By A.S.A. axiom of congruency)

$\therefore BG = DF$ (c.p.c.t.)

(iii) $EG = CF$ (c.p.c.t.) (Q.E.D.)

Question 14.

In each of the following diagrams, find the values of x and y .



Solution:



In $\triangle ABC$ and $\triangle ADE$

$AB = AD$ (Given)

$\angle BAC = \angle DAE$ (Given)

$AC = AE$ (Given)

$\therefore \triangle ABC \cong \triangle ADE$
(SAS)

$\therefore \triangle ABC \cong \triangle ADE$

(By S.S.S. rule of congruence)

$\therefore BC = DE$ (Corresponding sides)

Now, $\angle BAC = 120^\circ$ and $\angle DAE = 120^\circ$
and $\angle BAC + \angle DAE = 240^\circ$

$\therefore \angle BAC + \angle DAE + \angle BAE + \angle CAD = 360^\circ$

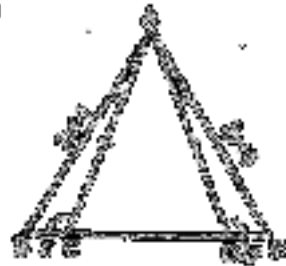
$240^\circ + \angle BAE + \angle CAD = 360^\circ$

$\therefore \angle BAE + \angle CAD = 360^\circ - 240^\circ = 120^\circ$

$\therefore \angle BAE + \angle CAD + \angle BAC + \angle DAE = 360^\circ$

$\therefore \angle BAE + \angle CAD + 120^\circ + 120^\circ = 360^\circ$

or



In $\triangle ABC$ and $\triangle ADE$

$AB = AD$ (Given)

$\angle BAC = \angle DAE$ (Given)

$AC = AE$ (Given)

$\therefore \triangle ABC \cong \triangle ADE$

(By S.S.S. rule of

congruence)

$\therefore BC = DE$ (Corresponding sides)

(Given)

(Given)

(Given)

(Given)

$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z} \Rightarrow \frac{y+x}{xy} = \frac{1}{z} \Rightarrow z(y+x) = xy$
 $\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$
 (Reciprocal of sum of reciprocals)
 $\Rightarrow \frac{1}{z} = \frac{y+x}{xy} \Rightarrow z = \frac{xy}{y+x}$
 Multiplying equation (1) by z and substituting equation (2) we get
 $\frac{1}{z} \times z = \frac{y+x}{xy} \times z$
 $1 = \frac{y+x}{xy} \times \frac{xy}{y+x}$
 $1 = 1$
 Hence, $z = \frac{xy}{y+x}$

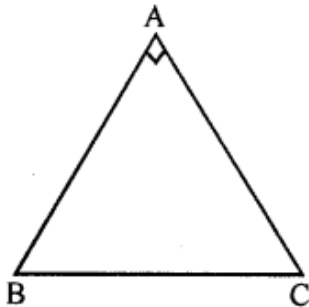
Exercise 10.3

Question 1.

ABC is a right angled triangle in which $\angle A = 90^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

In right $\triangle ABC$, $\angle A = 90^\circ$

$$\begin{aligned} \therefore \angle B + \angle C &= 180^\circ - \angle A \\ &= 180^\circ - 90^\circ = 90^\circ \end{aligned}$$



$$\therefore AB = AC$$

$$\therefore \angle C = \angle B \quad (\text{Angles opposite to equal sides})$$

$$\therefore \angle B + \angle B = 90^\circ \Rightarrow 2\angle B = 90^\circ$$

$$\therefore \angle B = \frac{90^\circ}{2} = 45^\circ$$

$$\therefore \angle B = \angle C = 45^\circ$$

Solution:

Question 2.

Show that the angles of an equilateral triangle are 60° each.

Solution:

ΔABC is an equilateral triangle

$$\therefore AB = BC = CA$$

$$\therefore \angle A = \angle B = \angle C \quad (\text{Opposite to equal sides})$$

$$\text{But } \angle A + \angle B + \angle C = 180^\circ$$

(Sum of angles of a triangle)

$$\therefore \angle A + \angle A + \angle A = 180^\circ$$

$$\Rightarrow 3\angle A = 180^\circ \Rightarrow \angle A = \frac{180^\circ}{3} = 60^\circ$$

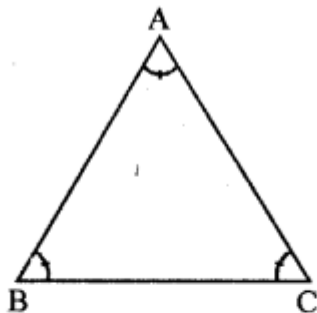
$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

Question 3.

Show that every equiangular triangle is equilateral.

Solution:

ΔABC is an equiangular



$$\therefore \angle A = \angle B = \angle C$$

In ΔABC

$$\because \angle B = \angle C$$

$$\therefore AC = AB \quad (\text{Sides opposite to equal angles})$$

...(i)

Similarly, $\angle C = \angle A$

$$\therefore BC = AB$$

...(ii)

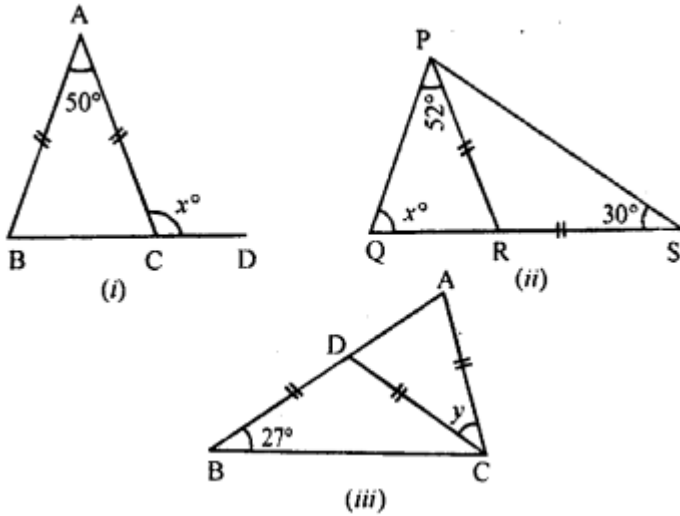
From (i) and (ii)

$$AB = BC = AC$$

$\therefore \Delta ABC$ is an equilateral triangle

Question 4.

In the following diagrams, find the value of x :



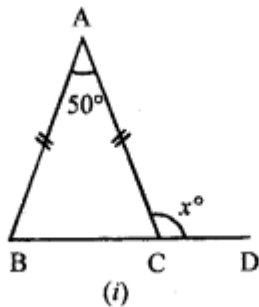
Solution:

(i) In following diagram given that $AB = AC$

i.e. $\angle B = \angle ACB$ (angles opposite to equal sides in a triangles are equal)

Now, $\angle A + \angle B + \angle ACB = 180^\circ$

(sum of all angles in a triangle is 180°)



$$\Rightarrow 50^\circ + \angle B + \angle B = 180^\circ$$

$$[\because \angle A = 50^\circ \text{ (given) } \angle B = \angle ACB]$$

$$\Rightarrow 50^\circ + 2\angle B = 180^\circ \Rightarrow 2\angle B = 180^\circ - 50^\circ$$

$$\Rightarrow 2\angle B = 130^\circ \Rightarrow \angle B = \frac{130}{2} = 65^\circ$$

$$\therefore \angle ACB = 65^\circ$$

$$\text{Also, } \angle ACB + x^\circ = 180^\circ \quad (\text{Linear pair})$$

$$65^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 180^\circ - 65^\circ$$

$$\Rightarrow x^\circ = 115^\circ$$

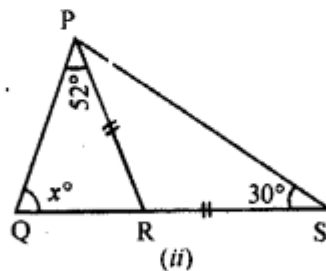
Hence, value of $x = 115$

(ii) In $\triangle PRS$,

Given that $PR = RS$

$$\therefore \angle PSR = \angle RPS$$

(angles opposite in a triangle, equal sides are equal)



$$\Rightarrow 30^\circ = \angle RPS \Rightarrow \angle RPS = 30^\circ \quad \dots (1)$$

$$\angle QPS = \angle QPR + \angle RPS$$

$$\Rightarrow \angle QPS = 52^\circ + 30^\circ$$

(Given, $\angle QPR = 52^\circ$ and from (1), $\angle RPS = 30^\circ$)

$$\Rightarrow \angle QPS = 82^\circ \quad \dots (2)$$

Now, in $\triangle PQS$

$$\angle QPS + \angle QSP + \angle PQS = 180^\circ$$

(sum of all angles in a triangles is 180°)

$$\Rightarrow 82^\circ + 30^\circ + x^\circ = 180^\circ$$

[From (2) $\angle QPS = 82^\circ$ and $\angle QSP = 30^\circ$ (given)]

$$\Rightarrow 112^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 180^\circ - 112^\circ$$

Hence, value of $x = 68$ Ans.

(iii) In the following figure, Given

that, $BD = CD = AC$ and $\angle DBC = 27^\circ$

Now, in $\triangle BCD$

$BD = CD$ (given)

$$\angle DBC = \angle BCD \dots (1)$$

(In a triangle sides opposite equal angles are equal)

Also, $\angle DBC = 27^\circ$ (given) (2)

From (1) and (2), we get

$$\angle BCD = 27^\circ \dots (3)$$

Now, ext. $\angle CDA = \angle DBC + \angle BCD$

[exterior angle is equal to sum of two interior opposite angles]

$$\Rightarrow \text{ext. } \angle CDA = 27^\circ + 27^\circ \text{ [From (2) and (3)]}$$

$$\Rightarrow \angle CDA = 54^\circ \dots (4)$$

In $\triangle ACD$,

$AC = CD$ (given)

$\angle CAD = \angle CDA$ (In a triangle, angles opposite to equal sides are equal)

$$\angle CAD = 54^\circ \text{ [From (4)]} \dots (5)$$

Also, in $\triangle ACD$

$$\angle CAD + \angle CDA + \angle ACD = 180^\circ$$

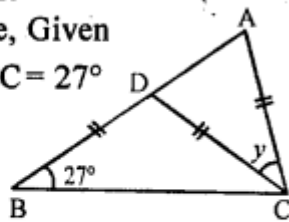
(sum of all angles in a triangle is 180°)

$$\Rightarrow 54^\circ + 54^\circ + y = 180^\circ \text{ [From (4) and (5)]}$$

$$\Rightarrow 108^\circ + y = 180^\circ \Rightarrow y = 180^\circ - 108^\circ$$

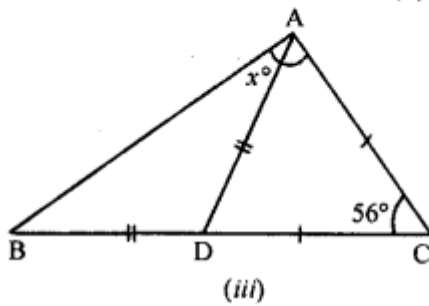
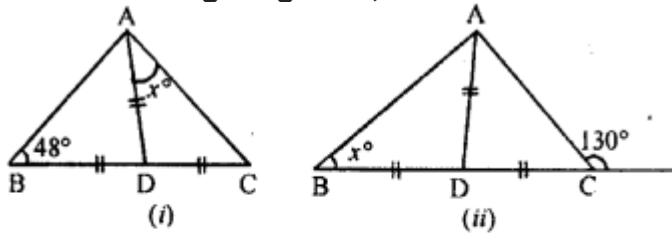
$$\Rightarrow y = 72^\circ$$

Hence, value of $y = 72^\circ$



Question 5.

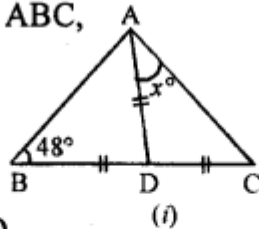
In the following diagrams, find the value of x :



Solution:

(i) In the following figure,

Given. In $\triangle ABC$,



$AD = BD = CD.$

$\angle B = 48^\circ, \angle DAC = x^\circ$

Now, in $\triangle ABD$

$BD = AD$ (given)

$\angle BAD = \angle B$ (1)

(angles opposite equal sides in a triangle are

equal).

$$\angle B = 48^\circ \quad \dots (2)$$

Now, in $\triangle ABD$

$$\text{From (1) and (2) } \angle BAD = 48^\circ \quad \dots\dots$$

(3)

$$\text{Exterior } \angle ADC = \angle B + \angle BAD$$

(In a triangle exterior angle is equal to sum of two interior opposite angles)

$$\angle ADC = 48^\circ + 48^\circ \Rightarrow \angle ADC = 96^\circ \quad \dots (4)$$

Now, in $\triangle ADC$

$$AD = DC \quad \text{(given)}$$

$$\therefore \angle C = \angle DAC \quad \dots (5)$$

(In a triangle, angles opposite equal sides are equal)

$$\angle DAC = x^\circ \quad \text{(given) } \dots (6)$$

From (5) and (6)

$$\angle C = x^\circ \quad \dots (7)$$

Now, in $\triangle ADC$

$$\angle C + \angle ADC + \angle DAC = 180^\circ$$

(sum of all the angles in a triangle is 180°)

$$\Rightarrow x^\circ + 96^\circ + x^\circ = 180^\circ \quad \text{[From 4, 6 and 7]}$$

$$\Rightarrow 2x^\circ = 180^\circ - 96^\circ \Rightarrow 2x^\circ = \frac{84}{2} \Rightarrow x^\circ = 42^\circ$$

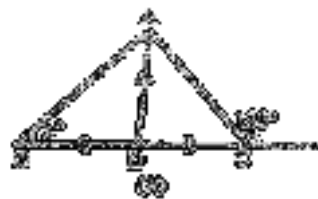
Hence, value of $x = 42$

(ii) Given in $\triangle ABC$,

Exterior $\angle ACE = 130^\circ$ and $AD = BD = DC$

To calculate the value of x .

$$\text{Now, } \angle ACD + \angle ACE = 180^\circ \quad \dots (1)$$



(1) $\triangle ABC$ is an isosceles triangle

$$\angle ADB = \angle ADC \quad \dots (1)$$

From (1) and (2)

$$\angle ADB + 90^\circ = \angle ADC + 90^\circ \Rightarrow \angle ADB = \angle ADC \quad \dots (2)$$

$$\Rightarrow \angle ADB = 90^\circ \quad \dots (3)$$

Now, in $\triangle ABC$,

$$AB = AC \quad \text{(Given)}$$

$$\therefore \angle B = \angle C \quad \dots (4)$$

(In a triangle, angles opposite equal sides are equal)

From (3) and (4)

$$\angle B = 90^\circ \quad \dots (5)$$

Now, in $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

(Sum of all angles in a triangle is 180°)

$$\angle A + 90^\circ + 90^\circ = 180^\circ \quad \text{[From (5)]}$$

$$\angle A + 180^\circ = 180^\circ \Rightarrow \angle A = 0^\circ \quad \dots (6)$$

$$\therefore \angle A = 0^\circ$$

$$\text{Hence, } \angle A = \angle B = \angle C = 90^\circ \quad \dots (7)$$

(In a triangle, all three angles are equal to each other)

(In a triangle, angles opposite equal sides are equal)

$$AB = AC \quad \text{(Given)}$$

$$\therefore \angle B = \angle C \quad \dots (8)$$

(In a triangle, angles opposite equal sides are equal)

From (7) and (8)

$$\angle A = \angle B = \angle C$$

$$\angle A = 2 \angle B \quad \dots (9)$$

From (8) and (9)

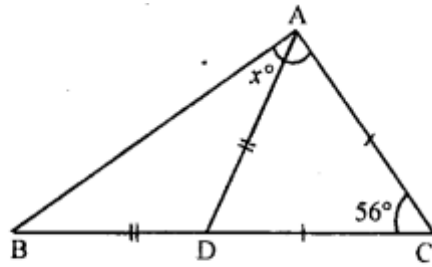
$$90^\circ = 2 \angle B \Rightarrow \angle B = 45^\circ$$

$$\text{[From (7) } \angle A = \angle B = \angle C \text{]}]$$

$$\Rightarrow \angle A = 90^\circ = \angle B = \angle C = 45^\circ$$

$$\text{Hence, } \angle A = \angle B = \angle C = 90^\circ$$

and in isosceles triangle,



(iii)

Given that $AC = CD$, $AD = BD$

and $\angle BAC = x^\circ$, $\angle ACD = 56^\circ$

to evaluate the value of x .

Now, in $\triangle ACD$

$AC = CD$ (given)

$\angle ADC = \angle DAC$ (1)

(In a triangle, angles opposite to equal sides are equal)

Also, $\angle ADC + \angle DAC + 56^\circ = 180^\circ$
(sum of all angles in a triangle is 180°)

$\Rightarrow \angle DAC + \angle DAC + 56^\circ = 180^\circ$

[From equation (1) $\angle ADC = \angle DAC$]

$\Rightarrow 2\angle DAC + 56^\circ = 180^\circ$

$\Rightarrow 2\angle DAC = 180^\circ - 56^\circ \Rightarrow 2\angle DAC = 124^\circ$

$\Rightarrow \angle DAC = \frac{124^\circ}{2} \Rightarrow \angle DAC = 62^\circ$ (2)

∴ $\angle ADC = 62^\circ$ (3)

[From (1) $\angle DAC = \angle ADC$]

Now, in $\triangle ABD$

$AD = BD$ (given)

$\angle ABD = \angle BAD$ (4)

(In a triangle, angles opposite equal sides are equal)

But ext. $\angle ADC = \angle ABD + \angle BAD$

$\Rightarrow 62^\circ = \angle BAD + \angle BAD$ [From (3) and (4)]

$\Rightarrow 62^\circ = 2\angle BAD \Rightarrow \angle BAD = 31^\circ$

$\Rightarrow \angle BAD = \frac{62^\circ}{2} \Rightarrow \angle BAD = 31^\circ$

∴ (5)

Now, from figure, $x^\circ = \angle BAD + \angle DAC$

$x^\circ = 31^\circ + 62^\circ \Rightarrow x^\circ = 93^\circ$ [From (4) and (5)]

∴ Hence, value of $x = 93$

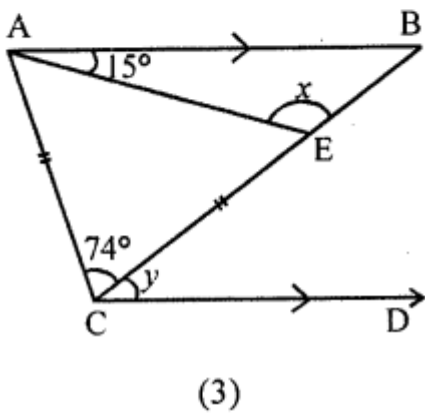
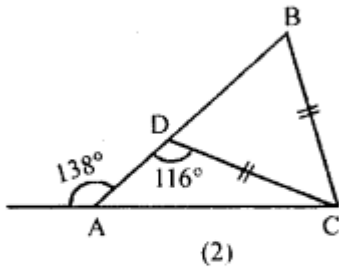
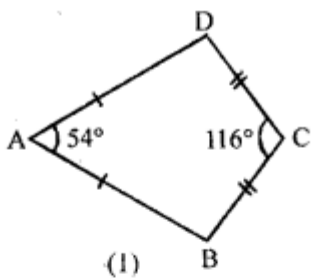
Question 6.

(a) In the figure (1) given below, $AB = AD$, $BC = DC$. Find $\angle ABC$.

(b) In the figure (2) given below, $BC = CD$. Find $\angle ACB$.

(c) In the figure (3) given below, $AB \parallel CD$ and $CA = CE$. If $\angle ACE = 74^\circ$ and $\angle BAE = 15^\circ$, find the values of x and y .

Solution:



Ex: In figure, $\angle A = 90^\circ$, $\angle B = 120^\circ$, $\angle C = 150^\circ$
 $\angle D = 90^\circ$ and $\angle E = 120^\circ$

To find $\angle F$



Construction Join AC

Now in $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ \quad \dots (1)$$

In a triangle, equal sides have equal angles opposite to them

$$\text{Here, } \angle A + \angle B + \angle C = 180^\circ$$

Since all the angles in a triangle is 180°

$$\rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle B + \angle C = 180^\circ \text{ (sum of angles in } \triangle ABC) \quad \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ - 120^\circ \Rightarrow \angle A + \angle C = 60^\circ$$

$$\Rightarrow \angle A + \angle C = \frac{60^\circ}{2} = 30^\circ \quad \dots (2)$$

Now in $\triangle ACF$

$$\angle A + \angle C + \angle F = 180^\circ \quad \dots (3)$$

In a triangle, equal sides have equal angles opposite to them

$$\text{Here, } \angle A + \angle C + \angle F = 180^\circ$$

Since all the angles in a triangle is 180°

$$\Rightarrow \angle A + \angle C + \angle F = 180^\circ$$

$$\angle A + \angle C + \angle F = 180^\circ \text{ (sum of angles in } \triangle ACF) \quad \angle A + \angle C + \angle F = 180^\circ$$

$$\Rightarrow \angle A + \angle C + \angle F = 180^\circ$$

$$\Rightarrow \angle A + \angle C + \angle F = 180^\circ - 60^\circ \Rightarrow \angle A + \angle C + \angle F = 120^\circ$$

$$\Rightarrow \angle A + \angle C + \angle F = \frac{120^\circ}{2} \Rightarrow \angle A + \angle C + \angle F = 60^\circ$$

$$\dots (4)$$

Now figure is a circle

$$\angle A + \angle C + \angle F = 180^\circ$$

$$\Rightarrow \angle A + \angle C + \angle F = 180^\circ - 60^\circ$$

Using (3) and (4)

$\Rightarrow \angle ABC = 90^\circ$

Q3 In following figure

Given $BC = CA$, $\angle ACB = 110^\circ$,

$\angle BCD = 100^\circ$,

Find $\angle ADE$.



Now, $\angle ABC + \angle ACB = 180^\circ$ [Linear pair]

$\Rightarrow 90^\circ + \angle ACB = 180^\circ$

$[\angle ACB = 110^\circ \text{ given}]$

$\Rightarrow \angle ACB = 180^\circ - 90^\circ$

$\Rightarrow \angle ACB = 90^\circ$

--- (1)

In $\triangle ABC$, $BC = CA$

[Given]

$\therefore \angle BAC = \angle ABC$

--- (2)

[In a triangle equal sides have equal angles opposite to them]

From (1) and (2)

$\angle BAC = 90^\circ$

--- (3)

Now, $\angle AED = \angle B + \angle ACD$

[Exterior angle is equal to sum of two interior opposite angles]

$\Rightarrow 100^\circ = 90^\circ + \angle ACD$

$[\angle ACD = 110^\circ \text{ given from (1), } \angle B = 90^\circ]$

$\Rightarrow 10^\circ + \angle ACD = 100^\circ$

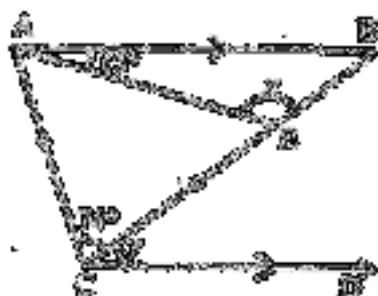
$\Rightarrow \angle ACD = 100^\circ - 10^\circ$

$\Rightarrow \angle ACD = 90^\circ$

Q4 In the figure, $AB \parallel CD$

$OA = OB$

$\angle AOE = 40^\circ$, $\angle BOE = 120^\circ$



In $\triangle AEC$
 $AC = CE$
 $\therefore \angle CAE = \angle CEA$
 But $\angle ACE = 74^\circ$
 $\therefore \angle CAE + \angle CEA = 180^\circ - 74^\circ = 106^\circ$
 $\therefore \angle CAE = \angle CEA = \frac{106^\circ}{2} = 53^\circ$
 Ext. $\angle AEB = \angle CAE + \angle ACE$
 $\Rightarrow x = 53^\circ + 74^\circ = 127^\circ$
 $\therefore AB \parallel CD$
 $\therefore \angle CAB + \angle ACD = 180^\circ$
 (Sum of cointerior angles)
 $\Rightarrow 15^\circ + 53^\circ + 74^\circ + y^\circ = 180^\circ$
 $\Rightarrow 142^\circ + y = 180^\circ$
 $\Rightarrow y = 180^\circ - 142^\circ = 38^\circ$

Question 7.

In $\triangle ABC$, $AB = AC$, $\angle A = (5x + 20)^\circ$ and each of the base angle is $\frac{2}{5}$ th of $\angle A$. Find the measure of $\angle A$.

Solution:

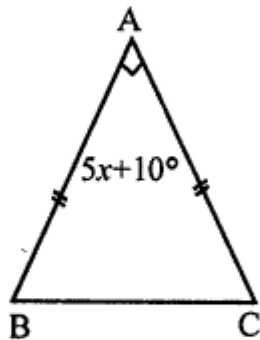
Given : In $\triangle ABC$, $AB = AC$

$$\angle A = (5x + 20)^\circ$$

$$\angle B = \angle C = \frac{2}{5}(\angle A)$$

$$= \frac{2}{5}(5x + 20)^\circ$$

$$= 2(x + 4)^\circ = 2x + 8$$



\therefore But sum of angles of a triangle = 180°

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 5x + 20 + 2x + 8 + 2x + 8 = 180^\circ$$

$$9x + 36 = 180^\circ$$

$$9x = 180 - 36 = 144$$

$$x = \frac{144}{9} = 16$$

$$\therefore \angle A = 5x + 20 = 5 \times 16 + 20$$

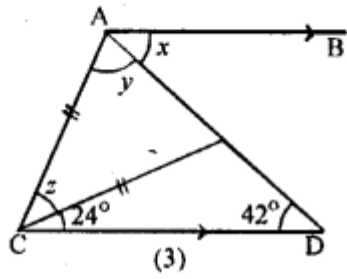
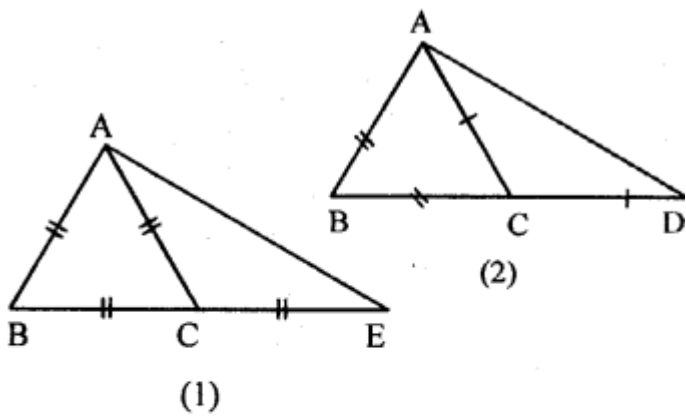
$$= 80^\circ + 20^\circ = 100^\circ$$

Question 8.

(a) In the figure (1) given below, ABC is an equilateral triangle. Base BC is produced to E, such that $BC' = CE$. Calculate $\angle ACE$ and $\angle AEC$.

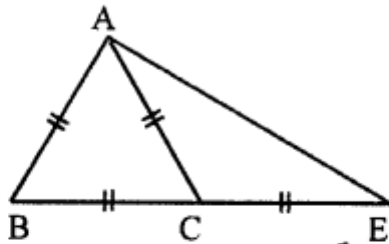
(b) In the figure (2) given below, prove that $\angle BAD : \angle ADB = 3 : 1$.

(c) In the figure (3) given below, $AB \parallel CD$. Find the values of x , y and \angle .



Solution:

(a) In following figure,
Given. ABC is an equilateral triangle $BC = CE$
To find. $\angle ACE$ and $\angle AEC$.



(1)

As given that ABC is an equilateral triangle,
 i.e. $\angle BAC = \angle B = \angle ACB = 60^\circ$ (1)
 (each angle of an equilateral triangle is 60°)

Now, $\angle ACE = \angle BAC + \angle B$
 (exterior angle is equal to sum of two interior
 opposite angles)

$$\Rightarrow \angle ACE = 60^\circ + 60^\circ \quad \text{[By (1)]}$$

$$\Rightarrow \angle ACE = 120^\circ$$

Now, in $\triangle ACE$

Given, $AC = CE$ ($\because AC = BC = CE$)

$$\angle CAE = \angle AEC \quad \text{.... (2)}$$

(In a triangle equal sides have equal angles
 opposite to them)

Also, $\angle CAE + \angle AEC + 120^\circ = 80^\circ$
(sum of all angles in a triangle is 180°)

$$\Rightarrow \angle AEC + \angle AEC + 120^\circ = 180 \quad [\text{By (2)}]$$

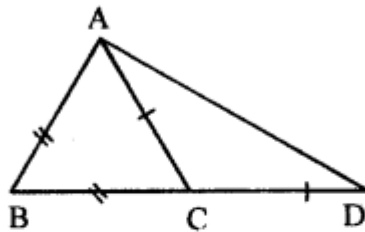
$$\Rightarrow 2\angle AEC = 180^\circ - 120^\circ \Rightarrow 2\angle AEC = 60^\circ$$

$$\Rightarrow 2\angle AEC = \frac{60^\circ}{2} = 30^\circ$$

Hence, $\angle ACE = 120^\circ$ and $\angle AEC = 30^\circ$

b) In following figure

Given. $\triangle ABD$, AC meets BD in C. $AB = BC$, $AC = CD$.



(2)

To prove. $\angle BAD : \angle ADB = 3 : 1$

Proof. In $\triangle ABC$,

$$AB = BC \quad (\text{Given})$$

$$\therefore \angle ACB = \angle BAC \quad \dots (1)$$

(In a triangle, equal angles opposite to them)

In $\triangle ACD$,

$$AC = CD \quad (\text{Given})$$

$$\therefore \angle ADC = \angle CAD$$

(In a triangle, equal sides have equal angles opposite to them)

$$\Rightarrow \angle CAD = \angle ADC \quad \dots (2)$$

From, Adding (1) and (2), we get

$$\angle BAC + \angle CAD = \angle ACB + \angle ADC$$

$$\angle BAD = \angle ACB + \angle ADC \quad \dots (3)$$

Now, in $\triangle ACD$

$$\text{Exterior } \angle ACB = \angle CAD + \angle ADC \quad \dots (4)$$

(In an triangle, exterior angle is equal to sum of two interior opposite angles)

$$\therefore \angle ACB = \angle ADC + \angle ADC$$

[From (2) and (4)]

$$\Rightarrow \angle ACB = 2 \angle ADC \quad \dots (5)$$

$$\text{Now, } \angle BAD = 2 \angle ADC + \angle ADC$$

[From (3) and (4)]

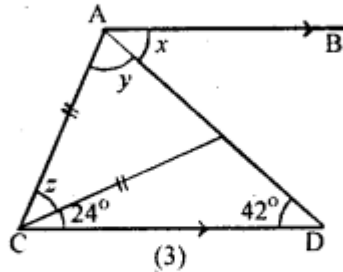
$$\Rightarrow \angle BAD = 3 \angle ADC \Rightarrow \frac{\angle BAD}{\angle ADC} = \frac{3}{1}$$

$$\Rightarrow \angle BAD : \angle ADC = 3 : 1 \quad (\text{Q.E.D.})$$

(c) In following figure,

Given. $AB \parallel CD$, $\angle ECD = 24^\circ$, $\angle CDE = 42^\circ$.

To find. The value of x , y and z .



Now, in $\triangle CDE$,

ext $\angle CEA = 24^\circ + 42^\circ$ [In a triangle exterior angle is equal to sum of two interior opposite angles]

$$\angle CEA = 66^\circ \quad \dots (1)$$

Now, in $\triangle ACE$

$$AC = CE \quad (\text{Given})$$

$$\therefore \angle CAE = \angle CEA$$

(In a triangle equal side have equal angles opposite to them)

$$y = 66^\circ \quad (\text{By equation (1)} \quad \dots(2))$$

$$\text{Also, } y + z + \angle CEA = 180^\circ$$

(sum of all angles in a triangle is 180°)

$$\Rightarrow 66^\circ + z + 66^\circ = 180^\circ$$

[From equation (1) and

(2)]

$$\Rightarrow z + 132^\circ = 180^\circ \Rightarrow z = 180^\circ - 132^\circ$$

$$\Rightarrow z = 48^\circ \quad \dots (3)$$

Given that, $AB \parallel CD$

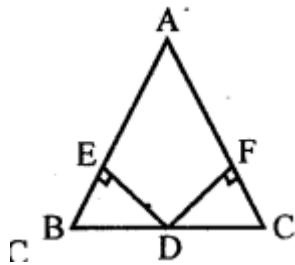
$$\therefore \angle x = \angle ADC \quad (\text{alternate angles})$$

$$x = 42^\circ \quad (4)$$

Hence, from (2), (3) and (4) equation gives $x = 42^\circ$, $y = 66^\circ$ and $z = 48^\circ$

Question 9.

In the given figure, D is mid-point of BC, DE and DF are perpendiculars to AB and AC respectively such that $DE = DF$. Prove that ABC is an isosceles triangle.

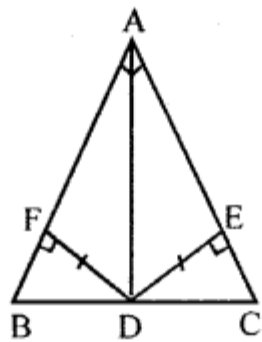


Solution:

Given : In $\triangle ABC$,
D is the mid-point of BC

$DE \perp AB, DF \perp AC$

$DE = DE$



To prove : $\triangle ABC$ is an isosceles triangle

Proof : In right $\triangle BED$ and $\triangle CDF$

Hypotenuse $BD = DC$ (D is mid-point)

Side $DF = DE$ (Given)

$\therefore \triangle BED \cong \triangle CDF$ (RHS axiom)

$\therefore \angle B = \angle C$

$\Rightarrow AB = AC$ (Sides opposite to equal angles)

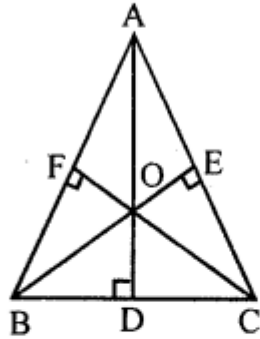
$\therefore \triangle ABC$ is an isosceles triangle

Question 10.

In the given figure, AD, BE and CF are altitudes of $\triangle ABC$. If $AD = BE = CF$, prove that $\triangle ABC$ is an equilateral triangle.

Solution:

Given : In the figure given,
 AD, BE and CF are altitudes of $\triangle ABC$ and
 $AD = BE = CF$



To prove : $\triangle ABC$ is an equilateral triangle

Proof : In the right $\triangle BEC$ and $\triangle BFC$

Hypotenuse $BC = BC$ (Common)

Side $BE = CF$ (Given)

$\therefore \triangle BEC \cong \triangle BFC$ (RHS axiom)

$\therefore \angle C = \angle B$ (c.p.c.t.)

$AB = AC$ (Sides opposite to equal angles)

...(i)

Similarly we can prove that $\triangle CFA \cong \triangle ADC$

$\therefore \angle A = \angle C$

$\therefore AB = BC$...(ii)

From (i) and (ii),

$AB = BC = AC$

$\therefore \triangle ABC$ is an equilateral triangle

Question 11.

In a triangle ABC , $AB = AC$, D and E are points on the sides AB and AC respectively such that $BD = CE$. Show that:

(i) $\triangle DBC \cong \triangle ECB$

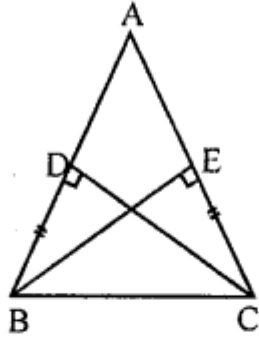
(ii) $\angle DCB = \angle ECB$

(iii) $OB = OC$, where O is the point of intersection of BE and CD .

Solution:

Given : In $\triangle ABC$, $AB = AC$

D and E are points on the sides AB and AC respectively such that $BD = CE$



To prove : (i) $\triangle DBC \cong \triangle ECB$

(ii) $\angle DCB = \angle ECB$

OB = OC where O is the point of intersection of BE and CD.

CD and BE are joined

Proof : In $\triangle ABC$, $AB = AC$
and $BD = CE$

$\therefore \angle C = \angle B$ (Opposite to equal sides)

In $\triangle DBC$ and $\triangle ECB$

$BC = BC$ (Common)

$BD = CE$ (Given)

$\angle B = \angle C$ (Proved)

(i) $\therefore \triangle DBC \cong \triangle ECB$ (SAS axiom)

(ii) $\therefore \angle DCB = \angle ECB$ (c.p.c.t.)

(iii) In $\triangle OBD$ and $\triangle OCE$

$\angle D = \angle E$ (each = 90°)

$DB = EC$ (given)

$\angle DOB = \angle EOC$ (vertically opposite

angles)

$\therefore \triangle OBD \cong \triangle OCE$ (A.A.S. Axiom)

$\therefore OB = OC$ (c.p.c.t.)

Question 12.

ABC is an isosceles triangle in which $AB = AC$. P is any point in the interior of

$\triangle ABC$ such that $\angle ABP = \angle ACP$. Prove that

(a) $BP = CP$

(b) AP bisects $\angle BAC$.

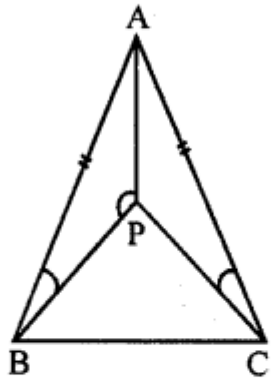
Solution:

Given : In an isosceles $\triangle ABC$, $AB = AC$

P is any point inside the $\triangle ABC$ such that

$\angle ABP = \angle ACP$

To prove : (a) $BP = CP$



(b) AP bisects $\angle BAC$

Proof : In $\triangle APB$ and $\triangle APC$

$AP = AP$ (Common)

$AB = AC$ (Given)

$\angle ABP = \angle ACP$ (Given)

$\therefore \triangle APB \cong \triangle APC$ (SSA axiom)

(i) $\therefore BP = CP$ (c.p.c.t.)

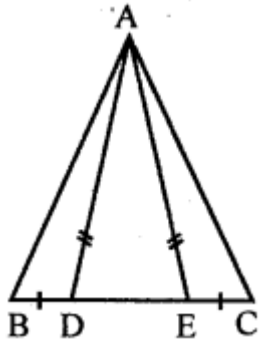
and $\angle BAP = \angle CAP$ (c.p.c.t.)

$\therefore AP$ bisects $\angle BAC$

Question 13.

In the adjoining figure, D and E are points on the side BC of $\triangle ABC$ such that $BD = EC$ and $AD = AE$. Show that $\triangle ABD \cong \triangle ACE$.

Solution:



Given : In the given figure, D and E are the points on the sides BC of $\triangle ABC$,

$BD = EC$ and $AD = AE$

To prove : $\triangle ABD \cong \triangle ACE$

Proof : \because In $\triangle ADE$

$\angle ADE = \angle AED$

$\therefore \angle AED = \angle ADE$

But $\angle ADE + \angle ADB = 180^\circ$ (Linear pair)

and $\angle AED + \angle AEC = 180^\circ$ (Linear pair)

$\therefore \angle ADB = \angle AEC$ ($\because \angle ADE = \angle AED$)

Now in $\triangle ABD$ and $\triangle ACE$

$AD = AE$ (Given)

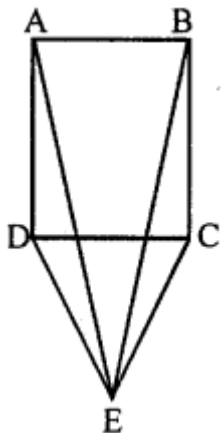
$BD = CE$ (Given)

$\angle ADB = \angle AEC$ (Proved)

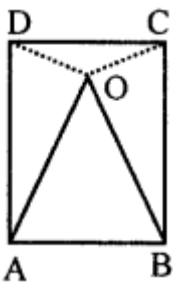
$\therefore \triangle ABD \cong \triangle ACE$ (SAS axiom)

Question 14.

(a) In the figure (i) given below, CDE is an equilateral triangle formed on a side CD of a square ABCD. Show that $\triangle ADE \cong \triangle BCE$ and hence, AEB is an isosceles triangle.



(b) In the figure (ii) given below, O is a point in the interior of a square $ABCD$ such that OAB is an equilateral triangle. Show that OCD is an isosceles triangle.



Solution:

(a) **Given :** In the figure, CDE is an equilateral triangle on the side CD of square ABCD

AE and BE are joined

To prove : (i) $\triangle ADE \cong \triangle BCE$

(ii) $\triangle AEB$ is an isosceles triangle.

Proof : \because Each angle of a square is 90° and each angle of an equilateral triangle is 60°

$$\begin{aligned}\therefore \angle ADE &= \angle ADC + \angle CDE \\ &= 90^\circ + 60^\circ = 150^\circ\end{aligned}$$

$$\text{Similarly, } \angle BCE = 90^\circ + 60^\circ = 150^\circ$$

Now in $\triangle ADE$ and $\triangle BCE$

$$AD = BC \quad (\text{Sides of a square})$$

$$DE = CE \quad (\text{Sides of an equilateral triangle})$$

$$\angle ADE = \angle BCE \quad (\text{Each } 150^\circ)$$

$$(i) \therefore \triangle ADE \cong \triangle BCE \quad (\text{SAS axiom})$$

(ii) $\therefore AE = BE$

Now in $\triangle AEB$,

$$AE = BE \quad (\text{Proved})$$

$\therefore \triangle AEB$ is an isosceles triangle

(b) **Given :** In the figure, O is a point in interior of the square ABCD such that OAB is an equilateral triangle.

To prove : $\triangle OCD$ is an isosceles triangle

Proof : $\because \triangle OAB$ is an equilateral triangle

$$\therefore OA = OB = AB$$

$$\angle OAD = \angle DAB - \angle OAB$$

$$= 90^\circ - 60^\circ = 30^\circ$$

Similarly, $\angle OBC = 30^\circ$

Now in $\triangle OAD$ and $\triangle OBC$

$OA = OB$ (Sides of equilateral triangle)

$AD = BC$ (Sides of a square)

$\angle OAD = \angle OBC$ (Each = 30°)

$\therefore \triangle OAD \cong \triangle OBC$ (SAS axiom)

$\therefore OD = OC$ (c.p.c.t.)

Now in $\triangle OCD$,

$OD = OC$

$\therefore \triangle OCD$ is an isosceles triangle

Question 15.

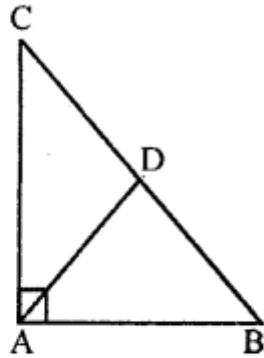
In the given figure, ABC is a right triangle with $AB = AC$. Bisector of $\angle A$ meets BC at D . Prove that $BC = 2AD$.

Solution:

In the given figure, $\triangle ABC$ is a right angled triangle, right angle at A

$AB = AC$

Bisector of $\angle A$ meets BC at D



To prove : $BC = 2AD$

Proof : In right $\triangle ABC$, $\angle A = 90^\circ$ and $AB = AC$

$$\therefore \angle B = \angle C = \frac{90^\circ}{2} = 45^\circ \quad (\because \angle B + \angle C = 90^\circ)$$

\therefore AD is bisector of $\angle A$

$$\therefore \angle DAB = \angle DAC = \frac{90^\circ}{2} = 45^\circ$$

Now in $\triangle ADB$

$$\angle DAB = \angle B \quad (\text{Each } 45^\circ)$$

$$\therefore AD = DB \quad \dots(i)$$

Similarly we can prove that in $\triangle ADC$,

$$\angle DAC = \angle C = 45^\circ$$

$$\therefore AD = DC \quad \dots(ii)$$

Adding (i) and (ii),

Adding (i) and (ii),

$$AD + AD = DB + DC = BD + DC$$

$$\Rightarrow 2AD = BC$$

Hence $BC = 2AD$

Exercise 10.4

Question 1.

In $\triangle PQR$, $\angle P = 70^\circ$ and $\angle R = 30^\circ$. Which side of this triangle is longest? Give reason for your answer.

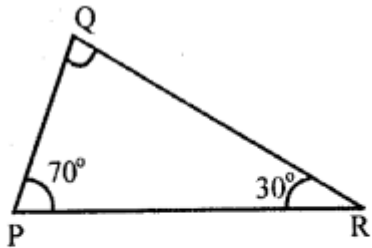
Solution:

In $\triangle PQR$, $\angle P = 70^\circ$, $\angle R = 30^\circ$

But $\angle P + \angle Q + \angle R = 180^\circ$

$$\Rightarrow 70^\circ + 30^\circ + \angle Q = 180^\circ$$

$$\Rightarrow 100^\circ + \angle Q = 180^\circ$$



$$\therefore \angle Q = 180^\circ - 100^\circ = 80^\circ$$

$\therefore \angle Q = 80^\circ$ the greatest angle

\therefore Its opposite side PR is the longest side

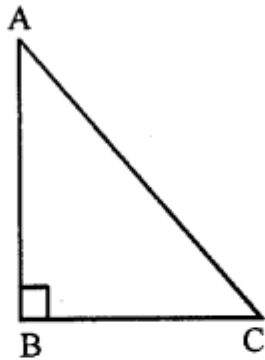
(Side opposite to greatest angle is longest)

Question 2.

Show that in a right angled triangle, the hypotenuse is the longest side.

Solution:

Given : In right angled $\triangle ABC$, $\angle B = 90^\circ$



To prove : AC is the longest side

Proof : In $\triangle ABC$,

$\therefore \angle B = 90^\circ$

$\therefore \angle A$ and $\angle C$ are acute angles
i.e., less than 90°

$\therefore \angle B$ is the greatest angle
or $\angle B > \angle C$ and $\angle B > \angle A$

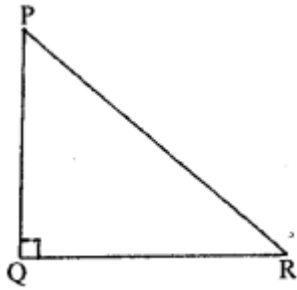
$\therefore AC > AB$ and $AC > BC$

Hence AC is the longest side

Question 3.

PQR is a right angle triangle at Q and $PQ : QR = 3:2$. Which is the least angle.

Solution:



Here, PQR is a right angle triangle at Q. Also given that

$$PQ : QR = 3 : 2$$

Let $PQ = 3x$, then, $QR = 2x$

It is clear that QR is the least side.

Then, we know that the least angle has least side opposite to it.

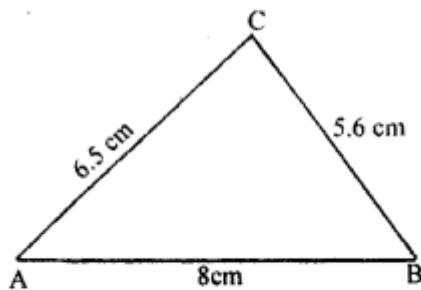
Hence, $\angle P$ is the least angle.

Question 4.

In $\triangle ABC$, $AB = 8$ cm, $BC = 5.6$ cm and $CA = 6.5$ cm. Which is (i) the greatest angle ?

(ii) the smallest angle ?

Solution:



Given that $AB = 8$ cm, $BC = 5.6$ cm, $CA = 6.5$ cm.

Here AB is the greatest side

Then $\angle C$ is the greatest angle

(\therefore the greater side has greater angle opposite to it)

Also, BC is the least side

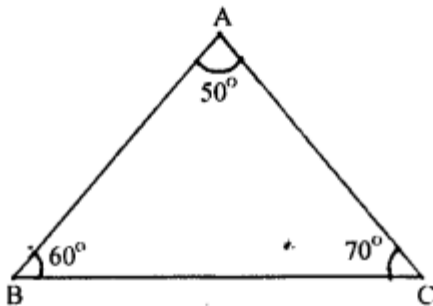
then $\angle A$ is the least angle

(\therefore the least side has least angle opposite to it)

Question 5.

In $\triangle ABC$, $\angle A = 50^\circ$, $\angle B = 60^\circ$, Arrange the sides of the triangle in ascending order.

Solution:



Given in a $\triangle ABC$,

$$\angle A = 50^\circ, \angle B = 60^\circ$$

$$\angle C = 180 - (\angle A + \angle B)$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow \angle C = 180^\circ - (50^\circ + 60^\circ)$$

$$\Rightarrow \angle C = 180^\circ - 110^\circ \Rightarrow \angle C = 70^\circ$$

Now, $\angle A < \angle B < \angle C$

$$BC < CA < AB$$

(\because greater angles has greater side opposite to it)

Hence, sides of $\triangle ABC$ in ascending order as BC, CA, AB.

Question 6.

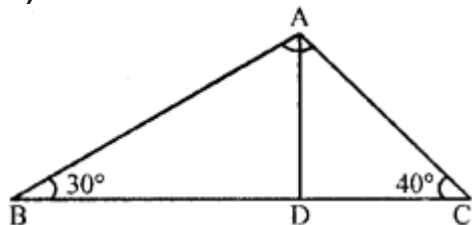
In figure given alongside, $\angle B = 30^\circ$, $\angle C = 40^\circ$ and the bisector of $\angle A$ meets BC at D. Show

(i) $BD > AD$

(ii) $DC > AD$

(iii) $AC > DC$

(iv) $AB > BD$



Solution:

Given : In $\triangle ABC$, $\angle B = 30^\circ$, $\angle C = 40^\circ$
and bisector of $\angle A$ meets BC at D

To prove :

- (i) $BD > AD$ (ii) $DC > AD$
(iii) $AC > DC$ (iv) $AB > BD$

Proof : In $\triangle ABC$,

$\angle B = 30^\circ$ and $\angle C = 40^\circ$

$$\therefore \angle BAC = 180^\circ - (30^\circ + 40^\circ) = 180^\circ - 70^\circ = 110^\circ$$

$\therefore AD$ is bisector of $\angle A$

$$\therefore \angle BAD = \angle CAD = \frac{110^\circ}{2} = 55^\circ$$

(i) Now in $\triangle ABD$,

$\therefore \angle BAD > \angle B$

$\therefore BD > AD$

(ii) In $\triangle ACD$,

$\angle CAD > \angle C$

$DC > AD$

(iii) $\angle ADC = 180^\circ - (40^\circ + 55^\circ) = 180^\circ - 95^\circ = 85^\circ$

In $\triangle ADC$,

$\therefore \angle ADC > \angle CAD$

$\therefore AC > DC$

(iv) Similarly,

$$\angle ADB = 180^\circ - \angle ADC = 180^\circ - 85^\circ = 95^\circ$$

\therefore In $\triangle ADB$

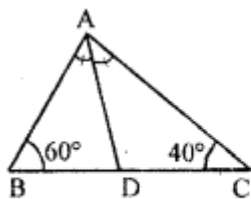
$AB > BD$

Hence proved.

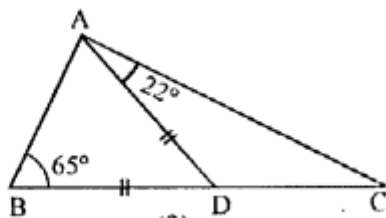
Question 7.

(a) In the figure (1) given below, AD bisects $\angle A$. Arrange AB , BD and DC in the descending order of their lengths.

(b) In the figure (2) given below, $\angle ABD = 65^\circ$, $\angle DAC = 22^\circ$ and $AD = BD$. Calculate $\angle ACD$ and state (giving reasons) which is greater : BD or DC ?



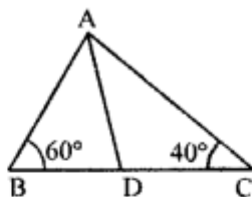
(1)



(2)

Solution:

(a) Given. In $\triangle ABC$, AD bisects $\angle A$,
 $\angle B = 60^\circ$ and $\angle C = 40^\circ$
 To arrange. AB, BD and DC in the descending
 order.



(1)

In $\triangle ABC$

$$\angle BAC + \angle B + \angle C = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow \angle BAC + 60^\circ + 40^\circ = 180^\circ$$

$$\text{[From given, } \angle B = 60^\circ, \angle C = 40^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 100^\circ \Rightarrow \angle BAC = 80^\circ$$

\therefore AD bisects $\angle A$

$$\therefore \angle BAD = \angle DAC = \frac{1}{2} \times \angle BAC$$

$$\Rightarrow \angle BAD = \angle DAC = \frac{1}{2} \times 80^\circ$$

$$\angle BAD = \angle DAC = 40^\circ \quad \dots (1)$$

In $\triangle ABD$, ext. $\angle ADC = \angle B + \angle BAD$

[In a triangle exterior angle is equal to sum of
 opposite interior angles]

$$\therefore \angle ADC = 60^\circ + 40^\circ$$

$$\Rightarrow \angle ADC = 100^\circ \quad \dots (2)$$

Similarly, In $\triangle ACD$, $\angle ADB$

$$= 40^\circ + 40^\circ = 80^\circ \quad \dots (3)$$

$$\text{Now, } \angle ADB = 80^\circ \quad \text{[From (3)]}$$

$$\angle BAD = 40^\circ \quad \text{[From (2)]}$$

$$\angle DAC = 40^\circ \quad \text{[From (1)]}$$

Now, $\angle ADB > \angle DAC = \angle BAD$

$$[\because 80^\circ > 40^\circ = 40^\circ]$$

Hence, AB, DC, BD in the descending order of
 their lengths

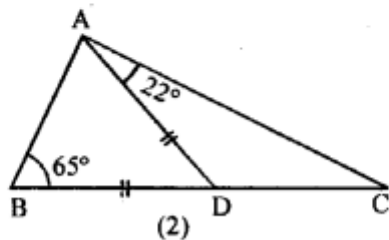
(Note : It can also written as AB, BD, DC in the

descending order $\because DC = BD$)

(b) Given. In $\triangle ABC$, $\angle ABD = 65^\circ$
 $\angle DAC = 22^\circ$, and $AD = BD$.

To calculate the $\angle ACD$ and say which is greater,
BD or DC.

Now, in $\triangle ABD$



$\therefore AD = BD$ (given)

$$\therefore \angle ABD = \angle BAD \quad \dots (1)$$

(In a triangle, equal sides have equal angles opposite to them)

$$\text{Also, } \angle ABD = 65^\circ \quad \dots (2)$$

From (1) and (2), we get

$$\angle BAD = 65^\circ \quad \dots (3)$$

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$(\angle BAD + \angle DAC) + \angle B + \angle C = 180^\circ$$

$$[\therefore \angle A = \angle BAD + \angle DAC]$$

$$\Rightarrow \angle BAD + \angle DAC + \angle B + \angle ACD = 180^\circ$$

$$[\therefore \angle C = \angle ACD]$$

$$\Rightarrow 65^\circ + 22^\circ + 65^\circ + \angle ACD = 180^\circ$$

(Substituting the value of $\angle BAD$, $\angle DAC$ & $\angle B$)

$$\Rightarrow 152^\circ + \angle ACD = 180^\circ$$

$$\Rightarrow \angle ACD = 180^\circ - 152^\circ \Rightarrow \angle ACD = 28^\circ$$

Now, $\angle BAD = 65^\circ$ [From (3)]

$$\text{and } \angle CAD = 22^\circ \quad (\text{Given})$$

$$\therefore \angle BAD > \angle CAD$$

$$\therefore BD > DC$$

[Greater angle has greater opposite side]

Hence, BD is greater than DC .

Question 8.

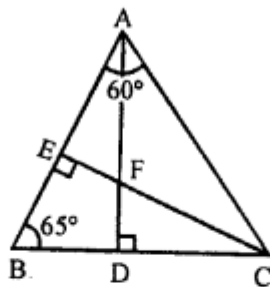
(a) In the figure (1) given below, prove that (i) $CF > AF$ (ii) $DC > DF$.

(b) In the figure (2) given below, $AB = AC$.

Prove that $AB > CD$.

(c) In the figure (3) given below, $AC = CD$. Prove that $BC < CD$.

(a) **Given.** In $\triangle ABC$, $AD \perp BC$, $CF \perp AB$.
 AD and CF intersect at F .
 $\angle BAC = 60^\circ$, $\angle ABC = 65^\circ$



(1)

To prove. (i) $CF > AF$ (ii) $DC > DF$

Proof. (i) In $\triangle AEC$,

$$\angle B + \angle BEC + \angle BCE = 180^\circ \quad \dots (1)$$

(sum of angles of a triangle = 180°)

$$\angle B = 65^\circ \quad (\text{Given}) \quad \dots (2)$$

$$\angle BEC = 90^\circ \quad [(CE \perp AB) \text{ Given}] \quad \dots (3)$$

Putting these value in equation (1), we get

$$65^\circ + 90^\circ + \angle BCE = 180^\circ$$

$$\Rightarrow 155^\circ + \angle BCE = 180^\circ \Rightarrow \angle BCE = 25^\circ$$

$$\Rightarrow \angle DCF = 25^\circ \quad [BCE = \angle DCF] \quad \dots (4)$$

Now in $\triangle CDF$,

$$\angle DCF + \angle FDC + \angle CFD = 180^\circ$$

Solution:

[sum of all angles in a triangle is 180°]

$$\Rightarrow 25 + 90^\circ + \angle CFD = 180^\circ$$

[From (4) $\angle DCF = 25^\circ$ & $AD \perp BC$, $\angle FDC = 90^\circ$]

$$\Rightarrow 115^\circ + \angle CFD = 180^\circ$$

$$\Rightarrow \angle CFD = 180^\circ - 115^\circ \quad \angle CFD = 65^\circ \dots (5)$$

Also, $\angle AFC + \angle CFD = 180^\circ$

[AFD is a straight line]

$$\Rightarrow \angle AFC + 65^\circ = 180^\circ$$

$$\Rightarrow \angle AFC = 180 - 65^\circ (\angle CFD = 65^\circ)$$

$$\Rightarrow \angle AFC = 115^\circ \dots (6)$$

Now, in ACE,

$$\angle ACE + \angle CEA + \angle BAC = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow \angle ACE + 90^\circ + 60^\circ = 180^\circ$$

[$\because \angle CEA = 90^\circ$, $\angle BAC = 60^\circ$]

$$\Rightarrow \angle ACE + 150^\circ = 180^\circ$$

$$\Rightarrow \angle ACE = 180^\circ - 150^\circ \Rightarrow \angle ACE = 30^\circ \dots (7)$$

Now, in ΔAFC ,

$$\angle AFC + \angle ACF + \angle FAC = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow 115^\circ + 30^\circ + \angle FAC = 180^\circ \text{ (By (6) and (7))}$$

$$\Rightarrow 145^\circ + \angle FAC = 180^\circ$$

$$\Rightarrow \angle FAC = 180^\circ - 145^\circ$$

$$\Rightarrow \angle FAC = 35^\circ \dots (8)$$

Now, in $\triangle AFC$,

$$\angle FAC = 35^\circ \quad [\text{From equation (8)}]$$

$$\angle ACF = 30^\circ \quad [\text{From equation (7)}]$$

$$\therefore \angle FAC > \angle ACF \quad (35^\circ > 30^\circ)$$

$$\therefore CF > AF$$

[Greater angle has greater side opposite to it]

Now, in $\triangle CDF$,

$$\angle DCF = 25^\circ \quad [\text{From equation (4)}]$$

$$\angle CFD = 65^\circ \quad [\text{From equation (5)}]$$

$$\therefore \angle CFD > \angle DCF \quad (\because 65^\circ > 25^\circ)$$

$$\therefore DC > DF.$$

[greater angle has greater side opposite to it]

(Q.E.D.)

(b) **Given.** In $\triangle ABD$, AC meets BD in C.

$\angle B = 70^\circ$, $\angle D = 40^\circ$ AB = AC.

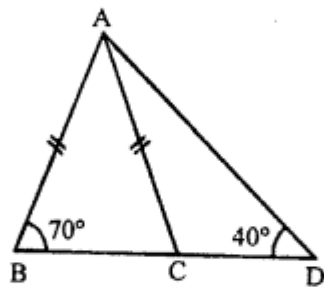
To prove. $AB > CD$.

Proof. In $\triangle ABC$,

AB = AC (given)

$\therefore \angle ACB = \angle B$ (1)

(In a triangle, equal sides have equal angles opposite to them)



(2)

Also, $\angle B = 70^\circ$ [Given] (2)

From (i) and (ii), we get

$\angle ACB + \angle ACD = 180^\circ$ [BCD is a st. line]

$\Rightarrow 70^\circ + \angle ACD = 180^\circ$ [From equation (3)]

$\Rightarrow \angle ACD = 180^\circ - 70^\circ$

$\Rightarrow \angle ACD = 110^\circ$ (4)

Now, in $\triangle ACD$,

$\angle CAD + \angle ACD + \angle D = 180^\circ$

[sum of all angles in a triangle is 180°]

$$\Rightarrow \angle CAD + 110^\circ + 40^\circ = 180^\circ \text{ [From (4)]}$$

$$\angle ACD = 110^\circ \text{ and } \angle D = 40^\circ \quad \text{(given)}$$

$$\Rightarrow \angle CAD + 150^\circ = 180^\circ$$

$$\Rightarrow \angle CAD = 180^\circ - 150^\circ$$

$$\Rightarrow \angle CAD = 30^\circ \quad \text{..... (5)}$$

Now, in $\triangle ACD$

$$\angle ACD = 110^\circ \quad \text{[From equation (4)]}$$

$$\angle CAD = 30^\circ \quad \text{[From equation (5)]}$$

$$\angle D = 40^\circ \quad \text{(given)}$$

$$\therefore \angle D > \angle CAD \quad (40^\circ > 30^\circ)$$

$$\therefore AC > CD$$

[Greater angle has greater side opposite to it]

$$\Rightarrow AB > CD \quad [\because AB = AC \text{ given}]$$

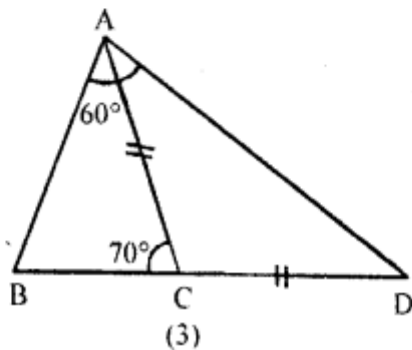
(Q.E.D.)

(c) Given. In $\triangle ACD$, $AC = CD$, $\angle BAD = 60^\circ$, $\angle ACB = 70^\circ$

To prove. $BC < CD$.

Proof. In $\triangle ACD$,

$$\therefore AC = CD, \quad \text{(Given)}$$



$$\therefore \angle CAD = \angle CDA \quad \dots (1)$$

[In a triangle If two sides are equal, then angles opposite to them are also equal]

$$\text{Also, } \angle ACB = 70^\circ \quad \dots (2)$$

$$\text{Now, } \angle ACB = \angle CAD + \angle CDA$$

[exterior angle is equal to sum of two interior opposite angles]

$$\Rightarrow 70^\circ = \angle CAD + \angle CAD$$

[From (1) and (2)]

$$\Rightarrow 70^\circ = 2\angle CAD$$

$$\Rightarrow 2\angle CAD = 70^\circ$$

$$\Rightarrow \angle CAD = \frac{70^\circ}{2} = 35^\circ$$

$$\therefore \angle BAD = 60^\circ \quad (\text{given})$$

$$\begin{aligned} \therefore \angle BAC &= \angle BAD - \angle CAD \\ &= 60^\circ - 35^\circ = 25^\circ \end{aligned}$$

$$\therefore \angle BAC < \angle CAD \quad [\because 25^\circ < 35^\circ]$$

$$\therefore BC < CD$$

[Greater angles has greater side opposite to it].

(Q.E.D.)

Question 9.

(a) In the figure (i) given below, $\angle B < \angle A$ and $\angle C < \angle D$. Show that $AD < BC$. (b) In the figure (ii) given below, D is any point on the side BC of $\triangle ABC$. If $AB > AC$, show that $AB > AD$.

Solution:

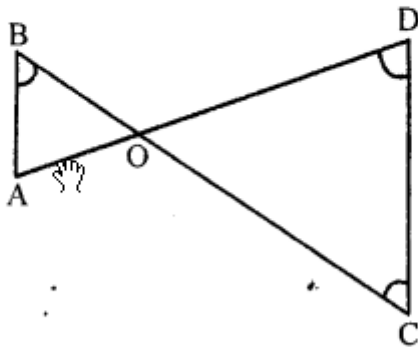
(a) In the given figure,
 $\angle B < \angle A$ and $\angle C < \angle D$

To prove : $AD < BC$

Proof : In $\triangle ABO$

$\angle B < \angle A$ (Given)

$\therefore AO < BO$... (i)



Similarly in $\triangle OCD$

$\angle C < \angle D$ (Given)

$$\therefore OD < OC$$

Adding (i) and (ii)

$$AO + OD < BO + OC$$

$$\Rightarrow AD < BC$$

Hence $AD < BC$

(b) In the given figure,

D is any point on BC of $\triangle ABC$

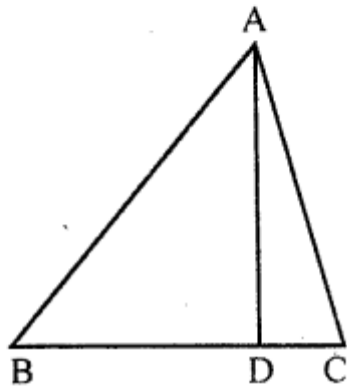
$$AB > AC$$

To prove : $AB > AD$

Proof : \because In $\triangle ABC$

$$AB > AC$$

$$\angle C > \angle B$$



In $\triangle ABD$
 Ext. $\angle ADC > \angle B$
 $\therefore \angle ADC > \angle C$
 $(\because \angle C > \angle B)$
 $\therefore AC > AD$...*(i)*
 But $AB > AC$
~~(Given)~~ ...*(ii)*
 \therefore From *(i)* and *(ii)*,
 $AB > AD$

Question 10.

- (i) Is it possible to construct a triangle with lengths of its sides as 4 cm, 3 cm and 7 cm? Give reason for your answer,
- (ii) Is it possible to construct a triangle with lengths of its sides as 9 cm, 7 cm and 17 cm? Give reason for your answer.
- (iii) Is it possible to construct a triangle with lengths of its sides as 8 cm, 7 cm and 4 cm? Give reason for your answer.

Solution:

(i) Length of sides of a triangle are 4 cm, 3 cm and 7 cm
 We know that sum of any two sides of a triangle is greater than its third side But $4 + 3 = 7$ cm
 Which is not possible
 Hence to construction of a triangle with sides 4 cm, 3 cm and 7 cm is not possible.

(ii) Length of sides of a triangle are 9 cm, 7 cm and 17 cm
 We know that sum of any two sides of a triangle is greater than its third side Now $9 + 7 = 16 < 17 \therefore$ It is not possible to construct a triangle with these sides.

(iii) Length of sides of a triangle are 8 cm, 7 cm and 4 cm We know that sum of any two sides of a triangle is greater than its third side Now $7 + 4 = 11 > 8$
 Yes, It is possible to construct a triangle with these sides.

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 18):

Question 1.

Which of the following is not a criterion for congruency of triangles?

- (a) SAS
- (b) ASA
- (c) SSA
- (d) SSS

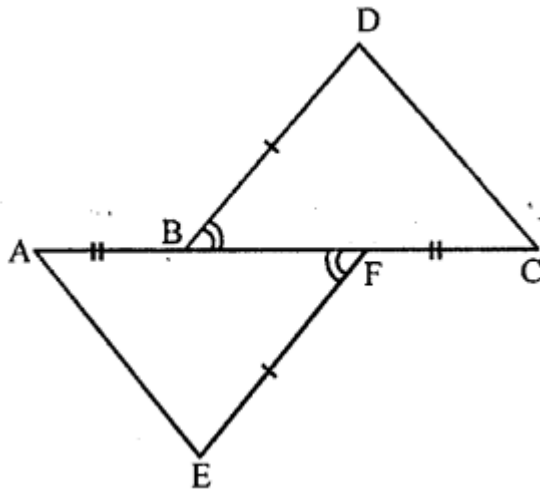
Solution:

Criteria of congruency of two triangles 'SSA' is not the criterion. (c)

Question 2.

In the adjoining figure, $AB = FC$, $EF = BD$ and $\angle AFE = \angle CBD$. Then the rule by which $\triangle AFE \cong \triangle CBD$ is

- (a) SAS
- (b) ASA
- (c) SSS
- (d) AAS



Solution:

In the figure given,

$\triangle AFE \cong \triangle CBD$ by SAS axiom

$$AB + BF = BF + FC \quad (\because AB = FC)$$

$$\Rightarrow AF = BC$$

$$EF = BD$$

$$\angle AFE = \angle CBD \quad (b)$$

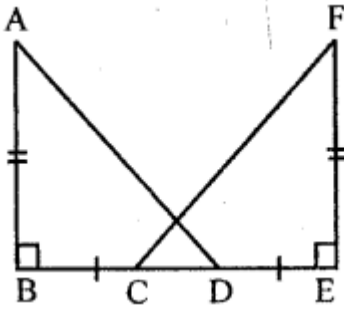
Question 3.

In the adjoining figure, $AB \perp BE$ and $FE \perp BE$. If $AB = FE$ and $BC = DE$, then

- (a) $\triangle ABD \cong \triangle EFC$
- (b) $\triangle ABD \cong \triangle FEC$
- (c) $\triangle ABD \cong \triangle ECF$
- (d) $\triangle ABD \cong \triangle CEF$

Solution:

In the figure given,



$AB \perp BE$ and $FE \perp BE$

$AB = FE, BC + CD = CD + DE$

($\because BC = DE$)

$\Rightarrow AB = FE$ and $BD = CE, \angle B = \angle E$

(Each 90°)

$\therefore \triangle ABD \cong \triangle FEC$

(b)

Question 4.

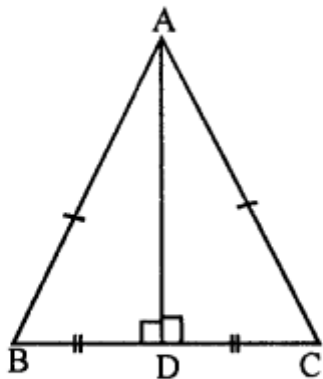
In the adjoining figure, $AB=AC$ and AD is median of $\triangle ABC$, then $\angle ADC$ is equal to

- (a) 60°
- (b) 120°
- (c) 90°
- (d) 75°

Solution:

In the given figure, $AB = AC$

AD is median of $\triangle ABC$



$\therefore D$ is mid-point $\Rightarrow BD = DC$

$\therefore AD \perp BC$

$\therefore \angle ADC = 90^\circ$

(c)

Question 5.

In the adjoining figure, O is mid point of AB. If $\angle ACO = \angle BDO$, then $\angle OAC$ is equal to

- (a) $\angle OCA$
- (b) $\angle ODB$
- (c) $\angle OBD$
- (d) $\angle BOD$

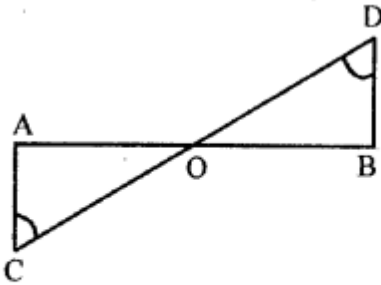
Solution:

In the given figure, O is mid-point of AB,

$$\angle ACO = \angle BDO$$

$$\angle AOC = \angle BOD$$

(Vertically opposite angles)



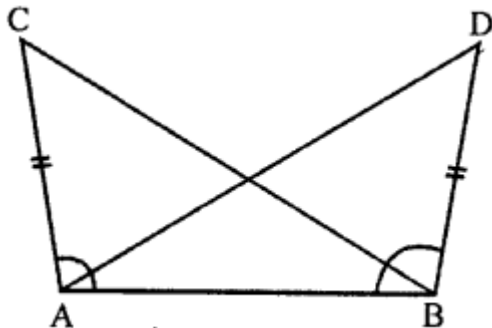
$$\therefore \triangle OAC \cong \triangle OBD \quad (\text{AAS})$$

$$\therefore \angle OAC = \angle OBD \quad (\text{c})$$

Question 6.

In the adjoining figure, $AC = BD$. If $\angle CAB = \angle DBA$, then $\angle ACB$ is equal to

- (a) $\angle BAD$
- (b) $\angle ABC$
- (c) $\angle ABD$
- (d) $\angle BDA$



Solution:

In the figure, $AC = BD$

$$\angle CAB = \angle DBA$$

$$AB = AB \quad \text{(Common)}$$

$$\therefore \triangle ABC \cong \triangle ABD \quad \text{(SAS axiom)}$$

$$\therefore \angle ACB = \angle BDA \quad \text{(c.p.c.t.) (d)}$$

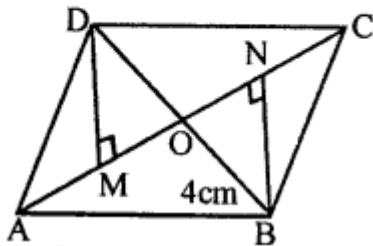
Question 7.

In the adjoining figure, ABCD is a quadrilateral in which BN and DM are drawn perpendiculars to AC such that $BN = DM$. If $OB = 4$ cm, then BD is

- (a) 6 cm
- (b) 8 cm
- (c) 10 cm
- (d) 12 cm

Solution:

In the given figure,
ABCD is a quadrilateral



$$BN \perp AC, DM \perp AC$$

$$BN = DM, OB = 4 \text{ cm}$$

In $\triangle ONB$ and $\triangle OMD$

$$BN = DM$$

$$\angle N = \angle M \quad \text{(Each } 90^\circ)$$

$$\angle BON = \angle DOM \text{ (Vertically opposite angles)}$$

$$\therefore \triangle ONB \cong \triangle OMD$$

$$\therefore OB = OD$$

$$\text{But } OB = 4 \text{ cm}$$

$$\therefore BD = BO + OD = 4 + 4 = 8 \text{ cm} \quad \text{(b)}$$

Question 8.

In $\triangle ABC$, $AB = AC$ and $\angle B = 50^\circ$. Then $\angle C$ is equal to

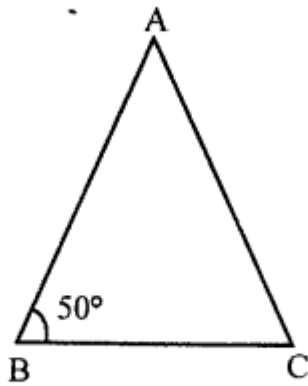
- (a) 40°
- (b) 50°

(c) 80°

(d) 130°

Solution:

In $\triangle ABC$, $AB = AC$



$\therefore \angle C = \angle B$ (Angles opposite to equal sides)

$\angle B = 50^\circ$

$\therefore \angle C = 50^\circ$

(b)

Question 9.

In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ is equal to

(a) 80°

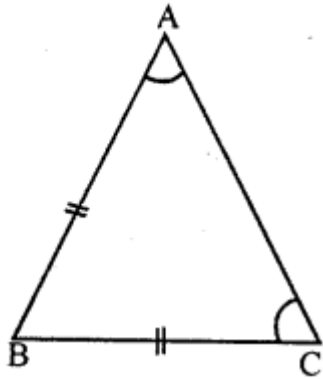
(b) 40°

(c) 50°

(d) 100°

Solution:

In $\triangle ABC = BC = AB$



$\therefore \angle A = \angle C$ (Angles opposite to equal sides)

$$\angle B = 80^\circ$$

$\therefore \angle A + \angle C = 180^\circ - 80^\circ = 100^\circ$

$$\text{But } \angle A = \angle C = 100^\circ$$

and $2\angle A$

$$\angle A = \frac{100^\circ}{2} = 50^\circ \quad (\text{c})$$

Question 10.

In $\triangle PQR$, $\angle R = \angle P$, $QR = 4$ cm and $PR = 5$ cm. Then the length of PQ is

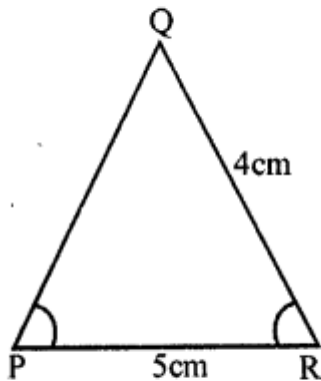
- (a) 4 cm
- (b) 5 cm
- (c) 2 cm
- (d) 2.5 cm

Solution:

In $\triangle PQR$

$\angle R = \angle P$, $QR = 4 \text{ cm}$

$PR = 5 \text{ cm}$



$$\therefore \angle P = \angle R$$

$$PQ = QR$$

\therefore (Sides opposite to equal angles

$$\therefore PQ = 4 \text{ cm}$$

(a)

Question 11.

In $\triangle ABC$ and $\triangle PQR$, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. The two triangles are

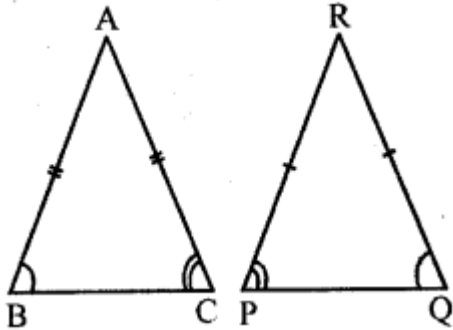
- (a) isosceles but not congruent
- (b) isosceles and congruent
- (c) congruent but isosceles
- (d) neither congruent nor isosceles

Solution:

In $\triangle ABC$ and $\triangle PQR$

$AB = AC, \angle C = \angle P$

$\angle B = \angle Q$



\therefore In $\triangle ABC, AB = AC$

$\angle C = \angle B$ (Opposite to equal sides)

But $\angle C = \angle P$ and $\angle B = \angle Q$

$\therefore \angle P = \angle Q$

$\therefore RQ = PR$

$\therefore \triangle RPQ$ is an isosceles triangle but not congruent (a)

Question 12.

Two sides of a triangle are of lengths 5 cm and 1.5 cm. The length of the third side of the triangle can not be

(a) 3.6 cm

(b) 4.1 cm

(c) 3.8 cm

(d) 3.4 cm

Solution:

In a triangle, two sides are 5 and 1.5 cm.

\therefore Sum of any two sides of a triangle is greater than its third side

\therefore Third side $< (5 + 1.5)$ cm

\Rightarrow Third side < 6.5 cm

or third side $+ 1.5 > 5$ cm

or third side $> 5 - 1.5 = 3.5$ cm

\therefore Third side cannot be equal to 3.4 cm (d)

Question 13.

If a, b, c are the lengths of the sides of a triangle, then

- (a) $a - b > c$
- (b) $c > a + b$
- (c) $c = a + b$
- (d) $c < a + b$

Solution:

a, b, c are the lengths of the sides of a triangle than $a + b > c$ or $c < a + b$
(Sum of any two sides is greater than its third side) **(d)**

Question 14.

It is not possible to construct a triangle when the lengths of its sides are

- (a) 6 cm, 7 cm, 8 cm
- (b) 4 cm, 6 cm, 6 cm
- (c) 5.3 cm, 2.2 cm, 3.1 cm
- (d) 9.3 cm, 5.2 cm, 7.4 cm

Solution:

We know that sum of any two sides of a triangle is greater than its third side $2.2 + 3.1 = 5.3 \Rightarrow 5.3 = 5.3$ is not possible **(c)**

Question 15.

In $\triangle PQR$, if $\angle R > \angle Q$, then

- (a) $QR > PR$
- (b) $PQ > PR$
- (c) $PQ < PR$
- (d) $QR < PR$

Solution:

In $\triangle PQR$, $\angle R > \angle Q$

$\therefore PQ > PR$ **(b)**

Question 16.

If triangle PQR is right angled at Q, then

- (a) $PR = PQ$
- (b) $PR < PQ$
- (c) $PR < QR$
- (d) $PR > PQ$

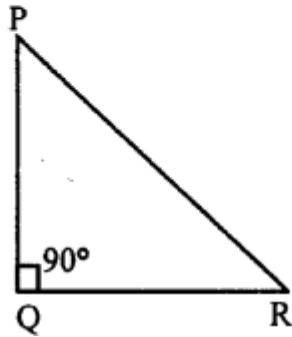
Solution:

In right angled ΔPQR ,

$$\angle Q = 90^\circ$$

Side opposite to greater angle is greater

$$\therefore PR > PQ \quad (d)$$



Question 17.

If triangle ABC is obtuse angled and $\angle C$ is obtuse, then

(a) $AB > BC$

(b) $AB = BC$

(c) $AB < BC$

(d) $AC > AB$

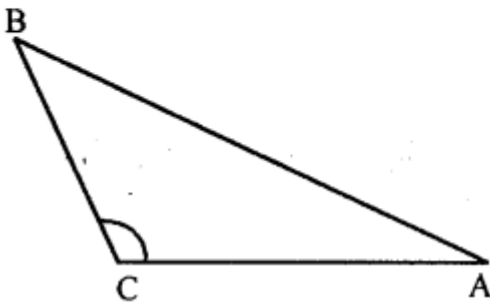
Solution:

In ΔABC , $\angle C$ is obtuse angle

$$AB > BC$$

(Side opposite to greater angle is greater)

(a)



Question P.Q.

A triangle can be constructed when the lengths of its three sides are

(a) 7 cm, 3 cm, 4 cm

(b) 3.6 cm, 11.5 cm, 6.9 cm

(c) 5.2 cm, 7.6 cm, 4.7 cm

(d) 33 mm, 8.5 cm, 49 mm

Solution:

We know that in a triangle, if sum of any two sides is greater than its third side, it is possible to construct it 5.2 cm, 7.6 cm, 4.7 cm is only possible. (c)

Question P.Q.

A unique triangle cannot be constructed if its

- (a) three angles are given
- (b) two angles and one side is given
- (c) three sides are given
- (d) two sides and the included angle is given

Solution:

A unique triangle cannot be constructed if its three angle are given, (a)

Question 18.

If the lengths of two sides of an isosceles are 4 cm and 10 cm, then the length of the third side is

- (a) 4 cm
- (b) 10 cm
- (c) 7 cm
- (d) 14 cm

Solution:

Lengths of two sides of an isosceles triangle are 4 cm and 10 cm, then length of the third side is 10 cm

(Sum of any two sides of a triangle is greater than its third side and 4 cm is not possible as $4 + 4 > 10$ cm.

Chapter Test

Question 1.

In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and $AB = EF$. Will the two triangles be congruent? Give reasons for your answer.

Solution:

In $\triangle ABC$ and $\triangle DEF$

$$\angle A = \angle D$$

$$\angle B = \angle E$$

$$AB = EF$$

In $\triangle ABC$, two angles and included side is given but in $\triangle DEF$, corresponding angles are equal but side is not included of there angle.

\therefore Triangles cannot be congruent.

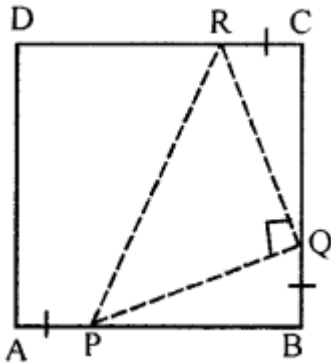
Question 2.

In the given figure, ABCD is a square. P, Q and R are points on the sides AB, BC and CD respectively such that $AP = BQ = CR$ and $\angle PQR = 90^\circ$. Prove that

(a) $\triangle PBQ \cong \triangle QCR$

(b) $PQ = QR$

(c) $\angle PRQ = 45^\circ$



Solution:

Given : In the given figure, ABCD is a square
P, Q and R are the points on the sides AB,
BC and CD respectively such that
 $AP = BQ = CR, \angle PQR = 90^\circ$

To prove : (a) $\triangle PBQ \cong \triangle QCR$

(b) $PQ = QR$

(c) $\angle PRQ = 45^\circ$

Proof : $\because AB = BC = CD$ (Sides of square)
and $AP = BQ = CR$ (Given)

Subtracting, we get

$$AB - AP = BC - BQ = CD - CR$$

$$\Rightarrow PB = QC = RD$$

Now in $\triangle PBQ$ and $\triangle QCR$

$$PB = QC \quad (\text{Proved})$$

$$BQ = CR \quad (\text{Given})$$

$$\angle B = \angle C \quad (\text{Each } 90^\circ)$$

$$\therefore \triangle PBQ \cong \triangle QCR \quad (\text{SAS axiom})$$

$$\therefore PQ = QR \quad (\text{c.p.c.t.})$$

$$\text{But } \angle PQR = 90^\circ \quad (\text{Given})$$

$$\angle RPQ = \angle PRQ$$

(Angles opposite to equal angles)

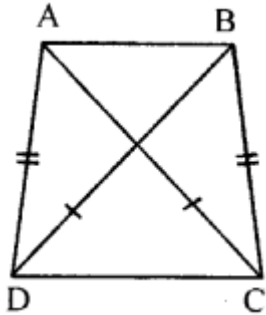
$$\text{But } \angle RPQ + \angle PRQ = 90^\circ$$

$$\angle RPQ = \angle PRQ = \frac{90^\circ}{2} = 45^\circ$$

Question 3.

In the given figure, $AD = BC$ and $BD = AC$. Prove that $\angle ADB = \angle BCA$.

Solution:



Given : In the figure,

$$AD = BC, BD = AC$$

To prove : $\angle ADB = \angle BCA$

Proof : In $\triangle ADB$ and $\triangle ACB$

$$AB = AB \quad \text{(Common)}$$

$$AD = BC \quad \text{(Given)}$$

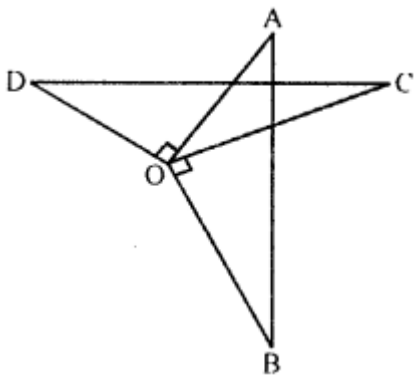
$$BD = AC \quad \text{(Given)}$$

$$\therefore \triangle ADB \cong \triangle ACB \quad \text{(SSS axiom)}$$

$$\therefore \angle ADB = \angle BCA \quad \text{(c.p.c.t.)}$$

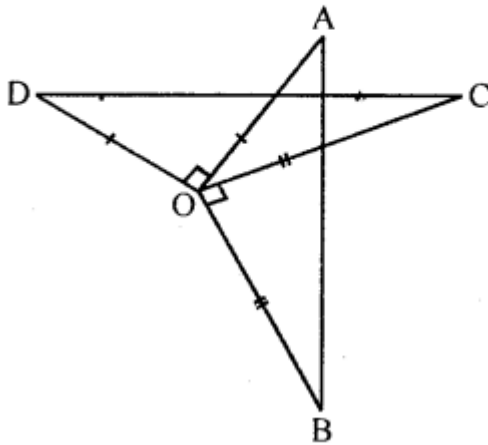
Question 4.

In the given figure, $OA \perp OD$, $OC \perp OB$, $OD = OA$ and $OB = OC$. Prove that $AB = CD$.



Solution:

Given : In the figure, $OA \perp OD$, $OC \perp OB$.
 $OD = OA$, $OB = OC$



To prove : $AB = CD$

Proof : $\angle AOD = \angle COB$ (each 90°)

Adding $\angle AOC$ (both sides)

$\angle AOD + \angle AOC = \angle AOC + \angle COB$

$\Rightarrow \angle COD = \angle AOB$

Now, in $\triangle AOB$ and $\triangle DOC$

$OA = OD$ (given)

$OB = OC$ (given)

$\angle AOB = \angle COD$ (proved)

$\therefore \triangle AOB \cong \triangle DOC$ (SAS axiom)

$\therefore AB = CD$ (c.p.c.t.)

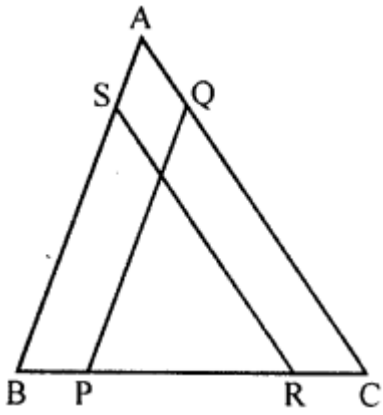
Question 5.

In the given figure, $PQ \parallel BA$ and $RS \parallel CA$. If $BP = RC$, prove that:

(i) $\triangle BSR \cong \triangle PQC$

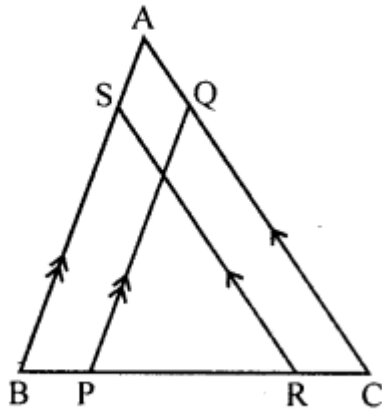
(ii) $BS = PQ$

(iii) $RS = CQ$.



Solution:

Given : In the given figure,
 $PQ \parallel BA$, $RS \parallel CA$
 $BP = RC$



To prove :

(i) $\triangle BSR \cong \triangle PQC$ (ii) $BS = PQ$

(iii) $RS = CQ$

Proof : $BP = RC$

$\therefore BC - RC = BC - BP$

$\therefore BR = PC$

Now, in $\triangle BSR$ and $\triangle PQC$

$\angle B = \angle P$ (corresponding angles)

$\angle R = \angle C$ (corresponding angles)

$BR = PC$ (proved)

$\therefore \triangle BSR \cong \triangle PQC$ (ASA axiom)

$\therefore BS = PQ$ (c.p.c.t.)

$RS = CQ$ (c.p.c.t.)

Question 6.

In the given figure, $AB = AC$, D is a point in the interior of $\triangle ABC$ such that $\angle DBC = \angle DCB$. Prove that AD bisects $\angle BAC$ of $\triangle ABC$.

Solution:

Given : In the figure given, $AB = AC$

D is a point in the interior of $\triangle ABC$

Such that $\angle DBC = \angle DCB$

To prove : AD bisects $\angle BAC$.

Construction : Join AD and produced it to BC in E

Proof : In $\triangle ABC$,

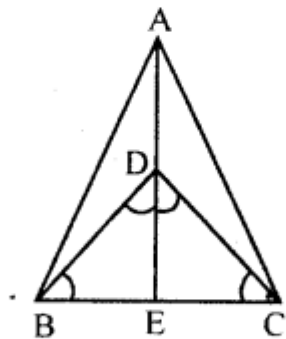
$AB = AC$

$\therefore \angle B = \angle C$ (Angles opposite to equal sides)

and $\angle DBC = \angle DCB$ (Given)

Subtracting, we get

$$\angle B - \angle DBC = \angle C - \angle DCB$$



$$\Rightarrow \angle ABD = \angle ACD$$

Now in $\triangle ABD$ and $\triangle ACD$

$AD = AD$ (Common)

$\angle ABD = \angle ACD$ (Proved)

$AB = AC$ (Given)

$\therefore \triangle ABD \cong \triangle ACD$ (SAS axiom)

$\therefore \angle BAD = \angle CAD$ (c.p.c.t.)

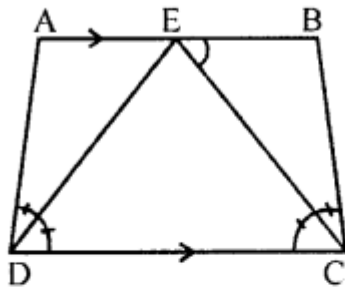
$\therefore AD$ is bisector of $\angle BAC$

Question 7.

In the adjoining figure, $AB \parallel DC$. CE and DE bisect $\angle BCD$ and $\angle ADC$ respectively. Prove that $AB = AD + BC$.

Solution:

Given : In the given figure, $AB \parallel DC$
CE and DE bisects $\angle BCD$ and $\angle ADC$
respectively



To prove : $AB = AD + BC$

Proof : $\because AD \parallel DC$ and ED is the transversal
 $\therefore \angle AED = \angle EDC$ (Alternate angles)
 $= \angle ADC$ ($\because ED$ is bisector of $\angle ADC$)
 $\therefore AD = AE$...*(i)*
(Sides opposite to equal angles)

Similarly,
 $\angle BEC = \angle ECD = \angle ECB$
 $\therefore BC = EB$...*(ii)*

Adding *(i)* and *(ii)*,
 $AD + BC = AE + EB = AB$
 $\therefore AB = AD + BC$

Question 8.

In $\triangle ABC$, D is a point on BC such that AD is the bisector of $\angle BAC$. CE is drawn parallel to DA to meet BD produced at E . Prove that $\triangle CAE$ is isosceles

Solution:

Given : In $\triangle ABC$,

D is a point on BC such that AD is the bisector of $\angle BAC$

$CE \parallel DA$ to meet BD produced at E

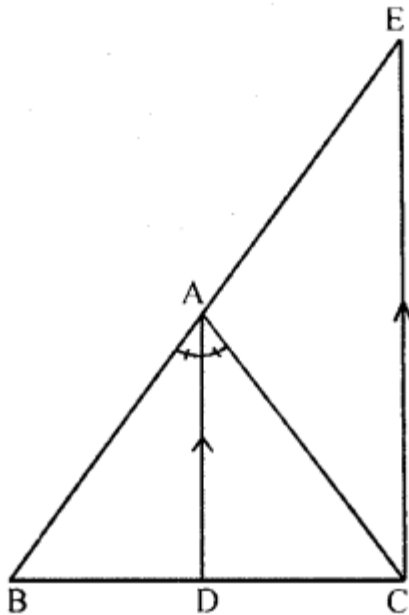
To prove : $\triangle CAE$ is an isosceles

Proof : $\because AD \parallel EC$ and AC is its transversal

$\therefore \angle DAC = \angle ACE$ (Alternate angles)

and $\angle BAD = \angle CEA$

(Corresponding angles)



But $\angle BAD = \angle DAC$

(\because AD is bisector of $\angle BAC$)

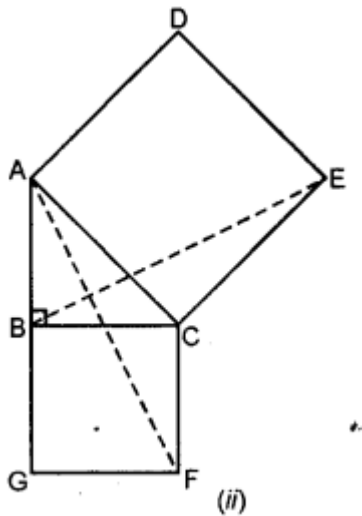
$\therefore \angle ACE = \angle CAE$

$AE = AC$ (Sides opposite to equal angles)

$\therefore \triangle ACE$ is an isosceles triangle.

Question 9.

In the figure (ii) given below, ABC is a right angled triangle at B, ADEC and BCFG are squares. Prove that $AF = BE$.



Solution:

Given. In right $\triangle ABC$, $\angle B = 90^\circ$
 ADEC and BCFG are squares on the sides AC and BC of $\triangle ABC$ respectively AF and BE are joined.

To prove. $AE = BE$

Proof. $\angle ACE = \angle BCF$

(each 90°)

Adding $\angle ACB$ both sides

$$\angle ACB + \angle ACE = \angle ACB + \angle BCF$$

$$\Rightarrow \angle BCE = \angle ACF$$

Now in $\triangle BCE$ and $\triangle ACF$,

$$CF = AC \text{ (sides of a square)}$$

$$BC = CF \text{ (sides of a square)}$$

$$\angle BCE = \angle ACF \text{ (proved)}$$

$$\therefore \triangle BCE \cong \triangle ACF \text{ (SAS postulate)}$$

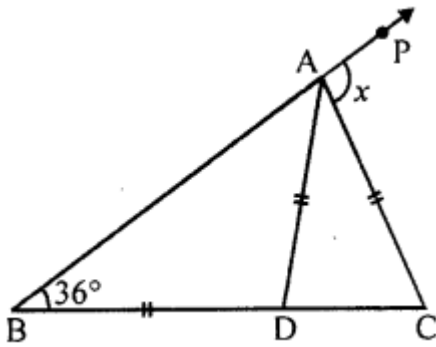
$$\therefore BE = AF$$

(c.p.c.t.)

Hence proved.

Question 10.

In the given figure, $BD = AD = AC$. If $\angle ABD = 36^\circ$, find the value of x .



Solution:

Given : In the figure, $BD = AD = AC$

$\angle ABD = 36^\circ$

To find : Measure of x .

Proof : In $\triangle ABD$,

$AD = BD$ (given)

$\therefore \angle ABD = \angle BAD = 36^\circ$ ($\because \angle ABD = 36^\circ$)

\therefore Ext. $\angle ADC = \angle ABD + \angle BAD$

$= 36^\circ + 36^\circ = 72^\circ$

But in $\triangle ADC$

$AD = AC$

$\therefore \angle ADC = \angle ACD = 72^\circ$

and Ext. $\angle PBC = \angle ABC + \angle ACD$

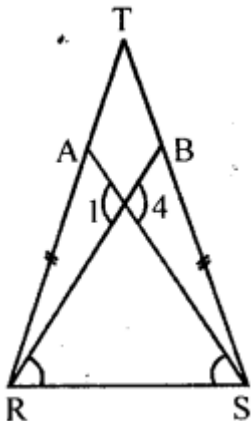
$= 36^\circ + 72^\circ = 108^\circ$

$\therefore x = 108^\circ$

(sum of interior opposite angles)

Question 11.

In the adjoining figure, $TR = TS$, $\angle 1 = 2\angle 2$ and $\angle 4 = 2\angle 3$. Prove that $RB = SA$.



Solution:

Given: In the figure, $\triangle RST$ is a triangle

$$TR = TS,$$

$$\angle 1 = 2\angle 2 \text{ and } \angle 4 = 2\angle 3$$

To prove : $RB = SA$

Proof : $\angle 1 = \angle 4$

$$\text{But } 2\angle 2 = \angle 1 \text{ and } 2\angle 3 = \angle 4$$

(Vertically opposite angles)

$$\therefore 2\angle 2 = 2\angle 3$$

$$\therefore \angle 2 = \angle 3$$

$$\therefore \text{But } \angle TRS = \angle TSR \quad (\because TR = TS \text{ given})$$

$$\therefore \angle TRS - \angle BRS = \angle TSR - \angle ASR$$

$$\Rightarrow \angle ARB = \angle BSA$$

Now in $\triangle RBT$ and $\triangle SAT$

$$\angle T = \angle T \quad (\text{Common})$$

$$TR = TS \quad (\text{Given})$$

$$\text{and } \angle TRB = \angle TSA \quad (\text{Proved})$$

$$\therefore \triangle RBT \cong \triangle SAT \quad (\text{SAS axiom})$$

$$\therefore RB = SA \quad (\text{c.p.c.t.})$$

Question 12.

(a) In the figure (1) given below, find the value of x .

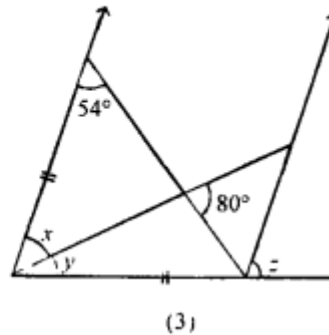
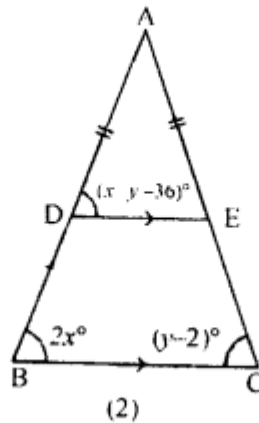
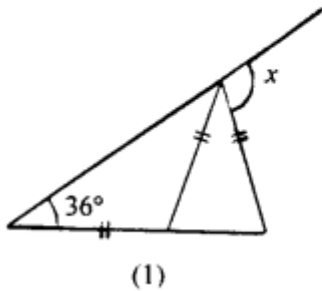
(b) In the figure (2) given below, $AB = AC$ and $DE \parallel BC$. Calculate

(i) x

(ii) y

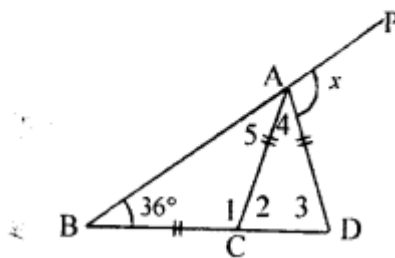
(iii) $\angle BAC$

(c) In the figure (3) given below, calculate the size of each lettered angle.



Solution:

(a) We have to calculate the value of x .



Now, in $\triangle ABC$

$$\angle 5 = 36^\circ \quad \dots (1)$$

Also, $36^\circ + \angle 1 + \angle 5 = 180^\circ$ [$\because AC = BC$]

[sum of all angles in a triangle is 180°]

$$\Rightarrow 36^\circ + \angle 1 + 36^\circ = 180^\circ \quad [\text{from (1)}]$$

$$\Rightarrow 72^\circ + \angle 1 = 180^\circ \Rightarrow \angle 1 = 180^\circ - 72^\circ$$

$$\Rightarrow \angle 1 = 108^\circ \quad \dots (2)$$

Also, $\angle 1 + \angle 2 = 180^\circ$ (Linear pair)

$$\Rightarrow 108^\circ + \angle 2 = 180^\circ \quad [\text{From (2)}]$$

$$\Rightarrow \angle 2 = 180^\circ - 108^\circ \Rightarrow \angle 2 = 72^\circ \quad \dots (3)$$

Also, $\angle 2 = \angle 3$ (AC = AD)

$$\therefore \angle 3 = 72^\circ \quad [\text{From (3)}] \dots (4)$$

Now, in $\triangle ACD$

$$\angle 2 + \angle 3 + \angle 4 = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow 72^\circ + 72^\circ + \angle 4 = 180^\circ \quad [\text{From (3) and (4)}]$$

$$\Rightarrow 144^\circ + \angle 4 = 180^\circ \Rightarrow \angle 4 = 180^\circ - 144^\circ$$

$$\Rightarrow \angle 4 = 36^\circ \quad \dots (5)$$

\therefore ABP is a St. line

$$\therefore \angle 5 + \angle 4 + x = 180^\circ$$

$$36^\circ + 36^\circ + x = 180^\circ \quad [\text{From (1) and (5)}]$$

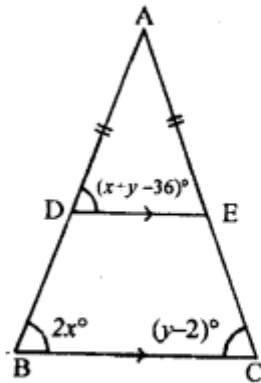
$$72^\circ + x = 180 \Rightarrow x = 108^\circ$$

Hence, value of $x = 108^\circ$ Ans.

(b) Given. $AB = AC$, and $DE \parallel BC$

$$\angle ADE = (x + y - 36)^\circ$$

$$\angle ABC = 2x^\circ \text{ and } \angle ACB = (y - 2)^\circ$$



To Calculate. (i) x (ii) y (iii) $\angle BAC$

Now, in $\triangle ABC$

$$\therefore AB = AC$$

$$2x = y - 2$$

[In a triangle equal sides here equal angle opposite to them]

$$2x - y = -2 \quad \dots (1)$$

$\therefore DE \parallel BC$,

$$x + y - 36 = 2x \quad \text{[corresponding angles]}$$

$$\Rightarrow x + y - 2x = 36 \Rightarrow -x + y = 36 \quad \dots (2)$$

From equation (1) and (2),

$$2x - y = -2$$

$$-x + y = 36$$

Adding,
$$\underline{x = 34}$$

Substituting the value of x in equation (1), we get

$$2 \times 34 - y = -2 \Rightarrow 68 - y = -2$$

$$\Rightarrow 68 + 2 = y \Rightarrow 70 = y \Rightarrow y = 70$$

Hence, value of $x = 34^\circ$

and value of $y = 70^\circ$

(iii) In $\triangle ABC$

$$\angle BAC + 2x^\circ + (y - 2)^\circ = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow \angle BAC + 2 \times 34^\circ + (70 - 2)^\circ = 180^\circ$$

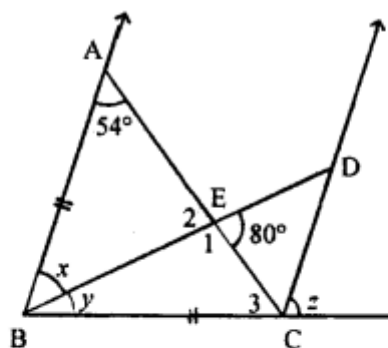
(Substituting the value of x and y)

$$\Rightarrow \angle BAC + 68^\circ + 68^\circ = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 136^\circ \Rightarrow \angle BAC = 44^\circ$$

Hence, value of $\angle BAC = 44^\circ$ **Ans.**

(c) **Given.** $\angle BAE = 54^\circ$, $\angle DEC = 80^\circ$ and $AB = BC$.



To calculate. The value of x , y and z .

Now $\angle 2 = 80^\circ$ (1)

(vertically opposite angles
 \therefore AC and BD cut at point E)

In $\triangle ABE$,

$54^\circ + x + \angle 2 = 180^\circ$
 (sum of all angles in triangle is 180°)

$\Rightarrow 54^\circ + x + 80^\circ = 180^\circ$ ($\because \angle 2 = 80^\circ$)

$\Rightarrow 134^\circ + x = 180^\circ \Rightarrow x = 180^\circ - 134^\circ$

$\Rightarrow x = 46^\circ$

Now, $\angle 1 + 80^\circ = 180^\circ$ (Linear pair)

$\angle 1 = 180^\circ - 80^\circ \Rightarrow \angle 1 = 100^\circ$ (2)

Also, $AB = BC$ (given)

$\angle 3 = 54^\circ$

(In a triangle equal sides have equal angles)

Now, in $\triangle ABC$

$54^\circ + (x + y) + \angle 3 = 180^\circ$
 (substituting the value of x and $\angle 3$)

$\Rightarrow 154^\circ + y = 180^\circ \Rightarrow y = 180^\circ - 154^\circ$

$\Rightarrow y = 26^\circ$ (3)

$\therefore AB \parallel CD$, $\therefore x + y = z$
 [corresponding angles]

$\Rightarrow 46^\circ + 26^\circ = z$ [From (2) and (3)]

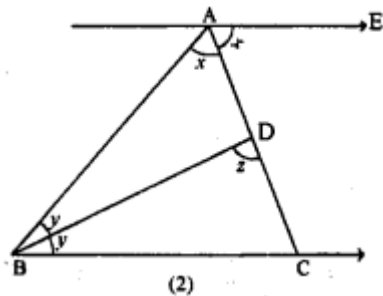
$\Rightarrow z = 46^\circ + 26^\circ \Rightarrow z = 72^\circ$

Hence, value of $x = 46^\circ$, $y = 26^\circ$

and $z = 72^\circ$

Question 13.

- (a) In the figure (1) given below, $AD = BD = DC$ and $\angle ACD = 35^\circ$. Show that
(i) $AC > DC$ (ii) $AB > AD$.
(b) In the figure (2) given below, prove that
(i) $x + y = 90^\circ$ (ii) $z = 90^\circ$ (iii) $AB = BC$



Solution:

(a) **Given :** In the figure given,

$$AD = BD = DC$$

$$\angle ACD = 35^\circ$$

To prove : (i) $AC > DC$, (ii) $AB > AD$

Proof : In $\triangle ADC$, $AD = DC$

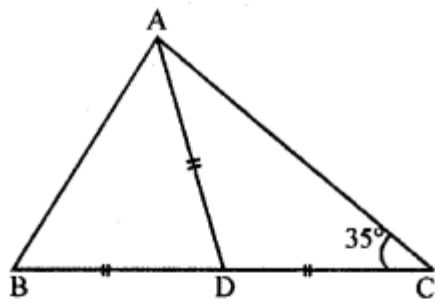
$$\therefore \angle DAC = \angle DCA = 35^\circ$$

$$\Rightarrow \angle ADC = 180^\circ - (\angle DAC + \angle DCA)$$

$$\therefore \angle ADC = 180^\circ - (35^\circ + 35^\circ)$$

$$= 180^\circ - 70^\circ = 110^\circ$$

$$\text{and Ext. } \angle ADB = \angle DAC + \angle DCA = 35^\circ + 35^\circ = 70^\circ$$



$$\therefore AD = BD$$

$$\angle BAD = \angle ABD$$

$$\text{But } \angle BAD + \angle ABD = 180^\circ - \angle ADB$$

$$\Rightarrow \angle ABD + \angle ABD = 180^\circ - 70^\circ = 110^\circ$$

$$\Rightarrow 2\angle ABD = 110^\circ \Rightarrow \angle ABD = \frac{110^\circ}{2} = 55^\circ$$

(i) Now $\therefore \angle ADC > \angle DAC$

$$\therefore AC > DC$$

$$\text{and } \angle ADB > \angle ABD$$

$$\therefore AB > AD$$

(b) Given. $\angle EAC = \angle BAC = x$

$$\angle ABD = \angle DBC = y$$

$$\angle BDC = z$$

To prove. (i) $x + y = 90^\circ$ (ii) $z = 90^\circ$

(iii) $AB = BC$

Proof. (i) $\therefore AE \parallel BC$

$$\therefore \angle ACB = x \quad [\text{Alternate angles}] \dots (1)$$

In $\triangle ABC$

$$x + (y + y) + \angle ACB = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow x + 2y + x = 180^\circ \quad [\text{From (1)}]$$

$$\Rightarrow 2x + 2y = 180^\circ$$

$$\Rightarrow 2(x + y) = 180^\circ \quad (\text{proved}) \dots (2)$$

$$\Rightarrow x + y = 90^\circ$$

(ii) Now, in $\triangle BCD$,

$$y + z + \angle BCD = 180^\circ$$

[sum of all angles in a triangle is 180°]

$$\Rightarrow y + z + x = 180^\circ$$

$$\Rightarrow 90^\circ + z = 180^\circ \quad [\text{From (2), } x + y = 90^\circ]$$

$$\Rightarrow z = 90^\circ \quad (\text{proved}) \dots (3)$$

(iii) In $\triangle ABC$

$$\angle BAC = \angle ABC = x \quad (\text{each same value})$$

$$\therefore AB = CB$$

(In a triangle equal angles has equal sides)
(proved)

Question 14.

In the given figure, $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC . If AD is extended to intersect BC at P , show that

(i) $\triangle ABD \cong \triangle ACD$

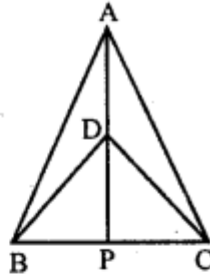
(ii) $\triangle ABP \cong \triangle ACP$

(iii) AP bisects $\angle A$ as well as $\angle D$

(iv) AP is the perpendicular bisector of BC .

Solution:

Given : In the figure, two isosceles triangles ABC and DBC are on the same base BC. With vertices A and D on the same side of BC.



AD is joined and produced to meet BC at P.

To prove :

- (i) $\triangle ABD \cong \triangle ACD$
- (ii) $\triangle ABP \cong \triangle ACP$
- (iii) AP bisects $\angle A$ as well as $\angle D$
- (iv) AP is the perpendicular bisector of BC

Proof : $\because \triangle ABC$ and $\triangle DBC$ are isosceles
 $AB = AC$ and $DB = DC$

- (i) Now in $\triangle ABD$ and $\triangle ACD$

$AB = AC$ (Proved)

$DB = DC$ (Proved)

$AD = AD$ (Common)

$\therefore \triangle ABD \cong \triangle ACD$ (SSS axiom)

$\therefore \angle BAD = \angle CAD$ (c.p.c.t.)

$\therefore ADP$ bisects $\angle A$
 and $\angle ADB = \angle ADC$ (c.p.c.t.)

But $\angle ADB + \angle BDP = \angle CAD + \angle CDP = 180^\circ$

$\therefore \angle BDP = \angle CDP$

$\therefore ADP$ bisects $\angle D$ also

Now in $\triangle APB$ and $\triangle APC$

$AB = AC$ (Given)

$AP = AP$ (Common)

and $\angle BAD = \angle CAD$ (Proved)

$\therefore \triangle APB \cong \triangle APC$ (SAS axiom)

$\therefore BP = CP$ (c.p.c.t.)

and $\angle APB = \angle APC$

But $\angle APB + \angle APC = 180^\circ$ (Linear pair)

$\therefore \angle APB = \angle APC = 90^\circ$

and $BP = PC$

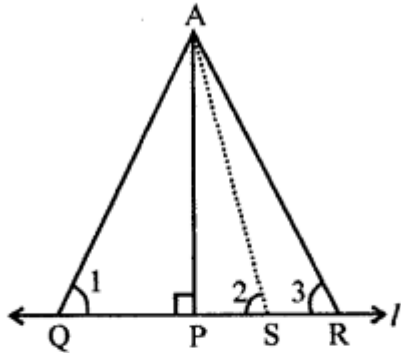
$\therefore AP$ is perpendicular bisector of BC

Question 15.

In the given figure, $AP \perp l$ and $PR > PQ$. Show that $AR > AQ$.

Solution:

Given : In the given figure,
 $AP \perp l$ and $PR > PQ$



To prove : $AR > AQ$

Construction : Take a point S on l,
Such that $PS = PQ$

Join A and S

Proof : In $\triangle AQP$ and $\triangle ASP$

$AP = AP$ (Common)

$QP = SP$ (Given)

$\angle APQ = \angle APS$ (Each 90°)

$\therefore \triangle AQP \cong \triangle ASP$ (SAS axiom)

$\therefore \angle 1 = \angle 2$

$AQ = AS$ (Sides opposite to equal angles)

In $\triangle ASR$

Ext. $\angle ASP > \angle ARS$

$\Rightarrow \angle 2 > \angle 3$

$\Rightarrow \angle 1 > \angle 3$ ($\because \angle 1 = \angle 2$)

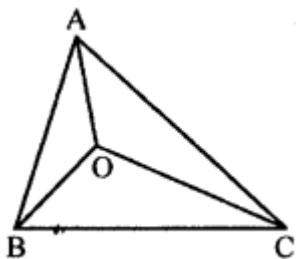
$\therefore AR > AQ$

Question 16.

If O is any point in the interior of a triangle ABC, show that

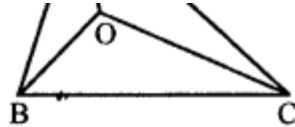
$$OA + OB + OC > \frac{1}{2}$$

$(AB + BC + CA)$.



Solution:

Given : In the figure,
O is any point in the
interior of $\triangle ABC$.



To prove : $OA + OB + OC > \frac{1}{2}(AB + BC + CA)$

Construct : Join B and C.

Proof : In $\triangle OBC$

$$OB + OC > BC \quad \dots(i)$$

(Sum of two sides of a triangle is greater than

its third side)

Similarly $OC + OA > CA$

and $OA + OB > AB$

Adding we get,

$$(OB + OC + OC + OA + OA + OB) > BC + CA + AB$$

$$\Rightarrow 2(OA + OB + OC) > AB + BC + CA$$

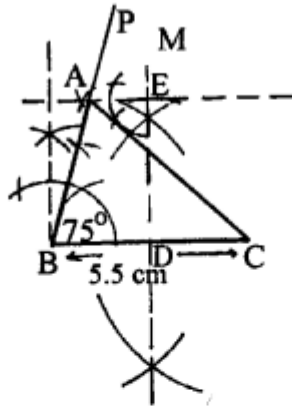
$$\Rightarrow OA + OB + OC > \frac{1}{2}(AB + BC + CA)$$

Question P.Q.

Construct a triangle ABC given that base $BC = 5.5$ cm, $\angle B = 75^\circ$ and height = 4.2 cm.

Solution:

Given. In a triangle ABC, Base $BC = 5.5$ cm, $\angle B = 75^\circ$ and height = 4.2 cm.

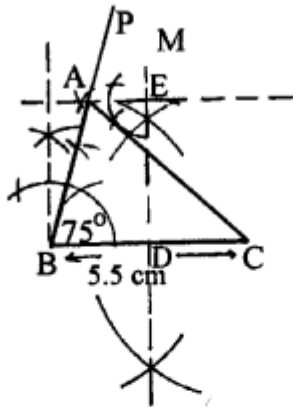


Required. To construct a triangle ABC.

Steps of Construction :

- (1) Draw a line $BC = 5.5$ cm.
- (2) Draw $\angle PBC = 75^\circ$.
- (3) Draw the perpendicular bisector of BC and cut the BC at point D .
- (4) Cut the DM at point E such that $DE = 4.2$ cm.
- (5) Draw the line at point which is parallel to line BC .
- (6) This parallel line cut the BP at point A .
- (7) Join AC .
- (8) ABC is the required triangle.

Given. In a triangle ABC, Base BC = 5.5 cm, $\angle B = 75^\circ$ and height = 4.2 cm.



Required. To construct a triangle ABC.

Steps of Construction :

- (1) Draw a line BC = 5.5 cm.
- (2) Draw $\angle PBC = 75^\circ$.
- (3) Draw the perpendicular bisector of BC and cut the BC at point D.
- (4) Cut the DM at point E such that DE = 4.2 cm.
- (5) Draw the line at point which is parallel to line BC.
- (6) This parallel line cut the BP at point A.
- (7) Join AC.
- (8) ABC is the required triangle.

Question P.Q.

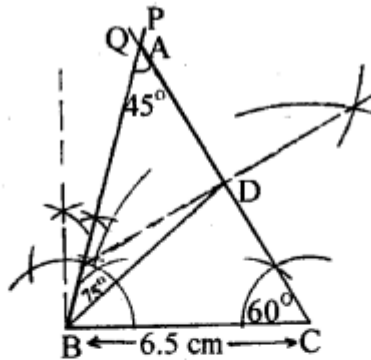
Construct a triangle ABC in which BC = 6.5 cm, $\angle B = 75^\circ$ and $\angle A = 45^\circ$. Also construct median of A ABC passing through B.

Solution:

Given. In $\triangle ABC$, $BC = 6.5$ cm, $\angle B = 75^\circ$ and $\angle A = 45^\circ$.

Required. (i) To construct a triangle ABC.

(ii) Construct median of $\triangle ABC$ passing through B.



Step of Construction.

- (1) Draw a line $BC = 6.5$ cm.
- (2) Make $\angle PBC = 75^\circ$.
- (3) Make $\angle BCQ = 60^\circ$.
- (4) BP and CQ cut at point A.
- (5) ABC is the required triangle.
- (6) Draw the bisector of AC.
- (7) The bisector line cut the line AC at point D.
- (8) Join BD.
- (9) BD is the required median of $\triangle ABC$ passing through B.

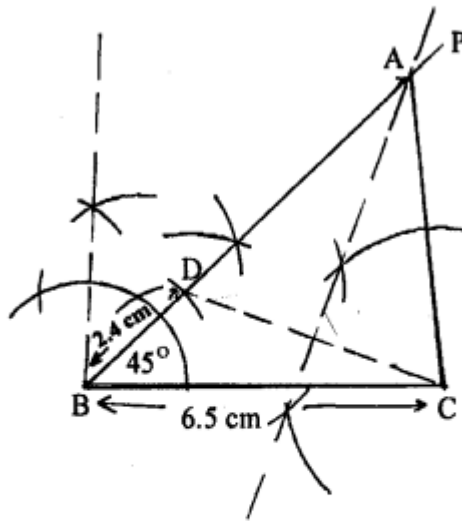
Question P.Q.

Construct triangle ABC given that $AB - AC = 2.4$ cm, $BC = 6.5$ cm. and $\angle B = 45^\circ$.

Solution:

Given. A triangle ABC in which $AB = AC = 2.4$ cm, $BC = 6.5$ cm, $\angle B = 45^\circ$.

Required. To construct a triangle ABC.



Steps of Construction :

- (1) Draw $BC = 6.5$ cm.
- (2) Draw BP making angle 45° with BC.
- (3) From BP, cut $BD = 2.4$ cm.
- (4) Join D and C.
- (5) Draw perpendicular bisector of DC which cuts BP at A.
- (6) Join A and C.
- (7) ABC is the required triangle.