Problems on Simultaneous Linear Equations

Question 1.

The sum of two numbers is 50 and their difference is 16. Find the numbers.

Solution:

According to first condition of the problem,

$x + y = 50$ ....(1)

According to second condition of the problem,

$x - y = 16$ .....(2)

From equation (1) and (2), we get

$$\begin{array}{r} x + y = 50 \\ x - y = 16 \\ \hline \text{Adding,}\quad 2x \quad = 66 \end{array}$$

$$\Rightarrow \quad x = \frac{66}{2} = 33$$

Substituting the value of x in equation (1), we get

$33 + y = 50 \Rightarrow y = 50 - 33 \Rightarrow y = 17$

Hence, two numbers are, 33 and 17. **Ans.**

Question 2.

The sum of two numbers is 2. If their difference is 20, find the numbers.

Solution:

Let two numbers are x and y

According to first condition of the problem,

$x + y = 50$ ....(1)

According to second condition of the problem,

$x - y = 16$ .....(2)

From equation (1) and (2), we get

$$\begin{array}{r} x + y = 50 \\ x - y = 16 \\ \hline 2x \quad = 66 \end{array}$$

Adding,

$$\Rightarrow \quad x = \frac{66}{2} = 33$$

Substituting the value of x in equation (1), we get

$33 + y = 50 \Rightarrow y = 50 - 33 \Rightarrow y = 17$

Hence, two numbers are, 33 and 17.

from equation (1) and equation (2), we get,

$$\begin{array}{r} x + y = 2 \\ x - y = 20 \\ \hline 2x \quad = 22 \end{array}$$

Adding,

$$\Rightarrow \quad x = \frac{22}{2} = 11$$

Substituting the value of x in equation (1), we get

$11 + y = 2 \Rightarrow y = 2 - 11 \Rightarrow y = -9$

Hence, the numbers are 11 and –9

Question 3.

The sum of two numbers is 43. If the larger is doubled and the smaller is tripled, the difference is 36. Find the two numbers.

Solution:

Let the two numbers are x and y

According to first condition of the problem,

$x + y = 43$ ....(1)

According to second condition of the problem,

$2x - 3y = 36$ .....(2)

Multiplying equation (1) by 3, we get

$3x + 3y = 129$ .....(3)

$2x - 3y = 36$ .....(2)

Adding, $\overline{5x = 165}$

$x = \frac{165}{5} \Rightarrow x = 33$

Substituting the value of x in equation (1), we get

$\Rightarrow 33 + y = 43 \Rightarrow y = 43 - 33 \Rightarrow y = 10$

Hence the two numbers are 33 and 10.

Question 4.

The cost of 5 kg of sugar and 7 kg of rice is Rs. 153, and the cost of 7 kg of sugar and 5 kg of rice is Rs. 147. Find the cost of 6 kg of sugar and 10 kg of rice.

Solution:

Let cost of 1 kg sugar = Rs. x

and let cost of 1 kg rice = Rs. y

Then cost of 5 kg of sugar = Rs. $5x$

cost of 7 kg of Rice = Rs. $7y$

cost of 7 kg of sugar = Rs. $7x$

cost of 5 kg of Rice = Rs. $5y$

According to first condition of the problem

$5x + 7y = 153$ .....(1)

According to second condition of the problem,

$7x + 5y = 147$ ....(2)

Multiplying equation (1) by 7 and equation (2) by 5 we get

$35x + 49y = 1071$ .....(3)

$35x + 25y = 735$ .....(4)

$- \quad - \quad -$

Subtracting, $24y = 336$

$$\Rightarrow \quad y = \frac{336}{24} \quad \Rightarrow \quad y = 14$$

Substituting the value of y in equation (1), we get

$\Rightarrow \quad 5x + 7 \times 14 = 153 \quad \Rightarrow \quad 5x + 98 = 153$

$\Rightarrow \quad 5x = 153 - 98 \quad \Rightarrow \quad 5x = 55$

$$\Rightarrow \quad x = \frac{55}{5} = \text{Rs. } 11$$

Hence, cost of 6 kg of sugar = Rs. 6×11 = Rs. 66

Cost of 10 kg of rice = Rs. 10×14 = Rs. 140

Then cost of 6 kg of sugar and 10 kg of rice

= Rs. 66 + Rs. 140 = Rs. 206

Question 5.

The class IX students of a certain public school wanted to give a farewell party to the outgoing students of class X. They decided to purchase two kinds of sweets, one costing Rs. 70 per kg and the other costing Rs. 84 per kg. They estimated that 36 kg of sweets were needed. If the total money spent on sweets was Rs. 2800, find how much sweets of each kind they purchased.

Solution:

Let sweets purchased x kg which cost Rs. 70 per kg.
and let sweets purchased y kg which cost Rs. 84 per kg
According to first condition of the problem

$$x + y = 36 \quad \text{...(1)}$$

According to second condition of the problem,

$$70x + 84y = 2800 \quad \text{....(2)}$$

Multiplying equation (1) by 70, we get

$$70x + 70y = 2520 \quad \text{....(3)}$$

$$70x + 84y = 2800 \quad \text{.....(2)}$$

Subtracting, $-14y = -280$

$$y = \frac{-280}{-14} = 20$$

Substituting the value of y in equation (1) we get

$$x + 20 = 36 \Rightarrow x = 36 - 20 = 16$$

Hence, sweets purchased 16 kg which cost Rs. 70 per kg
and sweets purchased 20 kg which cost Rs. 84 per kg

Question 6.
If from twice the greater of two numbers 16 is subtracted, the result is half the other number. If from half the greater number 1 is subtracted, the result is still half the other number. What are the numbers.
Solution:

Let the greater number $= x$

and let smaller number $= y$

According to first condition of the problem,

$$2x - 16 = \frac{y}{2} \Rightarrow 2x - \frac{y}{2} = 16 \Rightarrow \frac{4x - y}{2} = 16$$

$$\Rightarrow 4x - y = 32 \quad \text{....(1)}$$

According to second condition of the problem,

$$\frac{x}{2} - 1 = \frac{y}{2} \Rightarrow \frac{x}{2} - \frac{y}{2} = 1 \Rightarrow \frac{x - y}{2} = 1$$

$$\Rightarrow x - y = 2 \quad \text{....(2)}$$

From equation (1) and (2), we get

$$4x - y = 32 \quad \text{.....(1)}$$

$$x - y = 2 \quad \text{.....(2)}$$

$$- \quad + \quad -$$

Subtracting, $3x = 30$

$$x = \frac{30}{3} \Rightarrow x = 10$$

Substituting the value of x in equation (2), we get

$$\Rightarrow 10 - y = 2 \Rightarrow 10 - 2 = y$$

$$\Rightarrow 8 = y \Rightarrow y = 8$$

Hence, two numbers are 10, and 8

Question 7.

There are 38 coins in a collection of 20 paise coins and 25 paise coins. If the total value of the collection is Rs. 8.50, how many of each are there ?

Solution:

Let number of coins of 20 paise $= x$

and let number of coins of 25 paise $= y$

Then, according to first condition of given problem,

$x + y = 38$ ....(1)

according to second condition of given problem,

$20x + 25y = 850$ ...(2)

[$\because$ Rs. 8.50 = 850 paise]

Multiplying equation (1) by 20, we get

$20x + 20y = 760$ ....(3)

$20x + 25y = 850$ ....(2)

$- \quad - \quad -$

Subtracting, $-5y = -90$

$$\Rightarrow \quad y = \frac{-90}{-5} = 18$$

Substituting the value of y in equation (1), we get

$x + 18 = 38 \Rightarrow x = 38 - 18 \Rightarrow x = 20$

Hence, 20 paise 20 coins and 25 paise 18 coins

Question 8.

A man has certain notes of denominations Rs. 20 and Rs. 5 which amount to Rs. 380. If the number of notes of each kind is interchanged, they amount to Rs. 60 less as before. Find the number of notes of each denomination.

Solution:

Let number of 20 rupee notes = x

and number of 5 rupee notes = y

According to first condition of given problem,

$20x + 5y = 380$ ....(1)

According to second condition of given problem,

$5x + 20y = 380 - 60 \Rightarrow 5x + 20y = 320$ ....(2)

Multiplying equation (1) by 4, we get

$$\begin{array}{r} 80x + 20y = 1520 \\ 5x + 20y = 320 \\ - \quad - \quad - \\ \hline \text{Subtracting,} \quad 75x \quad = 1200 \end{array}$$

$$\Rightarrow x = \frac{1200}{75} \Rightarrow x = 16$$

Substituting the value of x in equation (1), we get

$20 \times 16 + 5y = 380 \Rightarrow 320 + 5y = 380$

$\Rightarrow 5y = 380 - 320 \Rightarrow 5y = 60$

$$\Rightarrow y = \frac{60}{5} = 12$$

Hence, Number of 20 rupee notes = 16

and 5 rupee notes = 12

Question 9.

The ratio of two numbers is $\frac{2}{3}$. If 2 is subtracted from the first and 8 from the second, the ratio becomes the reciprocal of the original ratio. Find the numbers.

Solution:

Let two numbers are x and y

Given ratio of two numbers is $= \frac{2}{3}$ reciprocal of the ratio is $= \frac{3}{2}$

According to first condition of given problem,

$$\frac{x}{y} = \frac{2}{3} \Rightarrow 3x = 2y \Rightarrow 3x - 2y = 0 \quad \text{....(1)}$$

According to second condition of given problem,

$$\frac{x-2}{y-8} = \frac{3}{2} \Rightarrow 2(x-2) = 3(y-8)$$

$$\Rightarrow 2x - 4 = 3y - 24 \Rightarrow 2x - 3y = -24 + 4$$

$$\Rightarrow 2x - 3y = -20 \quad \text{....(2)}$$

Multiplying equation (1) by 3 and equation (2) by 2, we get

$$9x - 6y = 0 \quad \text{....(3)}$$

$$4x - 6y = -40 \quad \text{....(4)}$$

$$- \quad + \quad +$$

Subtracting, $\overline{5x \qquad = 40}$

$$\Rightarrow x = \frac{40}{5} = 8$$

Substituting the value of x in equation (1) we get

$$3 \times 8 - 2y = 0 \Rightarrow 24 = 2y \Rightarrow 2y = 24$$

$$\Rightarrow y = \frac{24}{2} \Rightarrow y = 12$$

Hence, the numbers are 8 and 12

Question 10.

If 1 is added to the numerator of a fraction, it becomes $\frac{1}{5}$; if 1 is taken from the denominator, it becomes $\frac{1}{7}$, find the fraction.

Solution:

Let the fraction $= \frac{x}{y}$

According to given first condition of problem,

$$\frac{x+1}{y} = \frac{1}{5} \Rightarrow 5(x+1) = y \Rightarrow 5x+5 = y$$

$$\Rightarrow 5x - y = -5 \quad \text{....(1)}$$

According to second condition of given problem,

$$\frac{x}{y-1} = \frac{1}{7} \Rightarrow 7x = 1(y-1) \Rightarrow 7x = y-1$$

$$\Rightarrow 7x - y = -1 \quad \text{....(2)}$$

From equation (1) and (2), we get

$$\begin{array}{rl} & 5x - y = -5 \\ & 7x - y = -1 \\ - & + \quad + \\ \hline \text{Subtracting,} & -2x = -4 \end{array}$$

$$\Rightarrow x = \frac{-4}{-2} = 2$$

Substituting the value of x in equation (1), we get

$$5 \times 2 - y = -5 \Rightarrow 10 - y = -5 \Rightarrow -y = -5 - 10$$

$$\Rightarrow -y = -15 \Rightarrow y = 15$$

Hence, fraction $= \frac{x}{y} = \frac{2}{15}$

Question 11.

If the numerator of a certain fraction is increased by 2 and the denominator by 1, the fraction becomes equal to $\frac{5}{8}$ and if the numerator and denominator are each diminished by 1, the fraction becomes equal to $\frac{1}{2}$, find the fraction.

Solution:

Let the fraction $= \frac{x}{y}$

According to first condition of given problem,

$$\frac{x+2}{y+1} = \frac{5}{8} \Rightarrow 8(x+2) = 5(y+1)$$

$$\Rightarrow 8x + 16 = 5y + 5 \Rightarrow 8x - 5y = 5 - 16$$

$$\Rightarrow 8x - 5y = -11 \quad \text{....(1)}$$

According to second condition of given problem,

$$\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2(x-1) = 1\,(y-1)$$

$$\Rightarrow 2x - 2 = y - 1 \Rightarrow 2x - y = -1 + 2$$

$$\Rightarrow 2x - y = 1 \quad \text{....(2)}$$

Multiplying equation (2), by 5, we get

$$10x - 5y = 5$$

$$8x - 5y = -11$$

$$- \quad + \quad +$$

Subtracting, $2x = 16$

$$\Rightarrow x = \frac{16}{2} \Rightarrow x = 8$$

Substituting the value of x in equation (2), we get

$$2 \times 8 - y = 1 \Rightarrow 16 - y = 1 \Rightarrow -y = -15$$

$$\Rightarrow y = 15$$

Hence, fraction $= \frac{x}{y} = \frac{8}{15}$

Question 12.

Find the fraction which becomes $\frac{1}{2}$ when the denominator is increased by 4 and is equal to $\frac{1}{8}$, when the numerator is diminished by 5.

Solution:

Let the fraction $= \frac{x}{y}$

According to first condition of given problem,

$$\frac{x}{y+4} = \frac{1}{2} \Rightarrow 2x = 1(y+4)$$

$$\Rightarrow 2x = y+4 \Rightarrow 2x - y = 4 \quad \ldots(1)$$

According to second condition of given problem,

$$\frac{x-5}{y} = \frac{1}{8} \Rightarrow 8(x-5) = y \Rightarrow 8x - 40 = y$$

$$\Rightarrow 8x - y = 40 \quad \ldots(2)$$

From equation (1) and equation (2), we get

$$\begin{array}{r} 2x - y = 4 \\ 8x - y = 40 \\ - \quad + \quad - \\ \hline \text{Subtracting, } \quad -6x = -36 \end{array}$$

$$x = \frac{-36}{-6} = 6$$

Substituting the value of x in equation (1), we get

$$2 \times 6 - y = 4 \Rightarrow 12 - y = 4 \Rightarrow 12 - 4 = y$$

$$\Rightarrow 8 = y \Rightarrow y = 8$$

Hence, fraction $= \frac{6}{8}$

Question 13.

In a two digit number the sum of the digits is 7. If the number with the order of the digits reversed is 28 greater than twice the unit's digit of the original number, find the number.

Solution:

Let the digits at tens place $= x$

and let the digits at unit place $= y$

According to first condition of problem

$x + y = 7$ ....(1)

Also, number $= 10 \times x + y \times 1 = 10x + y$

reversing the number $= 10 \times y + x \times 1 = 10y + x$

According to second condition of problem,

$10y + x = 2y + 28 \Rightarrow 10y + x - 2y = 28$

$\Rightarrow x + 8y = 28$ ....(1)

From equation (1) and (2), we get

$$\begin{array}{r} x + y = 7 \\ x + 8y = 28 \\ - \quad - \quad - \\ \hline -7y = -21 \end{array}$$

Subtracting,

$$\Rightarrow y = \frac{-21}{-7} = 3$$

Substituting the value of y in equation (1), we get

$x + 3 = 7 \Rightarrow x = 7 - 3 \Rightarrow x = 4$

Hence, number $= 10 \times 4 + 3 = 40 + 3 = 43$

Question 14.

A number of two digits exceeds four times the sum of its digits by 6 and it is increased by 9 on reversing the digits. Find the number.

Solution:

Let the digit at ten's place $= x$

and let digit at unit place $= y$

Number $= 10 \times x + y \times 1 = 10x + y$

reversing the number $= 10 \times y + x \times 1 = 10y + x$

According to first condition of given problem,

$10x + y = 4(x + y) + 6 \Rightarrow 10x + y = 4x + 4y + 6$

$\Rightarrow 10x + y - 4x - 4y = 6 \Rightarrow 6x - 3y = 6$

$\Rightarrow 3(2x - y) = 6 \Rightarrow 2x - y = \frac{6}{3}$

$\Rightarrow 2x - y = 2$ ....(1)

According to second condition of given problem,

$10x + y + 9 = 10y + x \Rightarrow 10x + y - 10y - x = -9$

$\Rightarrow 9x - 9y = -9 \Rightarrow x - y = \frac{-9}{9}$

$\Rightarrow x - y = -1$ ....(2)

From equation (1) and (2), we get

$2x - y = 2$ ....(1)

$x - y = -1$ ....(2)

$- \quad + \quad +$

Subtracting, $x \quad = 3$

Substituting the value of x in equation (1), we get

$2 \times 3 - y = 2 \Rightarrow 6 - y = 2 \Rightarrow -y = 2 - 6$

$\Rightarrow -y = -4 \Rightarrow y = 4$

Hence, number $= 10 \times 3 + 4 = 30 + 4 = 34$

Question 15.

When a two digit number is divided by the sum of its digits the quotient is 8. If the ten's digit is diminished by three times the unit's digit the remainder is 1. What is the number ?

Solution:

Let the digit at ten's place $= x$

and let digit at unit place $= y$

number $= 10 \times x + y \times 1 = 10x + y$

According to first condition of given problem

$$\frac{10x+y}{x+y} = 8 \Rightarrow 10x + y = 8(x + y)$$

$$\Rightarrow 10x + y = 8x + 8y \Rightarrow 2x - 7y = 0 \quad \ldots(1)$$

According to second condition of given problem,

$$\Rightarrow x - 3y = 1 \quad \ldots(2)$$

Multiplying equation (2) by 2, we get

$$2x - 6y = 2 \quad \ldots(3)$$

$$2x - 7y = 0 \quad \ldots(1)$$

$$- \quad + \quad -$$

Subtracting, $y = 2$

Substituting the value of y in equation (1), we get

$$\Rightarrow 2x - 7 \times 2 = 0 \Rightarrow 2x = 14$$

$$\Rightarrow x = \frac{14}{2} \Rightarrow x = 7$$

Hence, number $= 10 \times 7 + 2 = 70 + 2 = 72$

Question 16.

The result of dividing a number of two digits by the number with digits reversed is $1\frac{3}{4}$. If the sum of digits is 12, find the number.

Solution:

Let the digit at ten's place $= x$

and let digit at unit place $= y$, number $= 10x + y$

reversing the number $= 10 \times y + x \times 1 = 10y + x$

According to first condition of given problem,

$$\frac{10x+y}{10y+x} = 1\frac{3}{4} \Rightarrow \frac{10x+y}{10y+x} = \frac{7}{4}$$

$$\Rightarrow 4(10x+y) = 7(10y+x) \Rightarrow 40x + 4y = 70y + 7x$$

$$\Rightarrow 40x + 4y - 70y - 7x = 0 \Rightarrow 33x - 66y = 0$$

$$\Rightarrow 33(x-2y) = 0 \Rightarrow x - 2y = 0 \quad \text{....(1)}$$

According to second condition of given problem,

$x + y = 12$ ....(2)

From equation (1) and (2), we get

$x - 2y = 0$ ....(1)

$x + y = 12$ ....(2)

$- \quad - \quad -$

Subtracting, $-3y = -12$

$$y = \frac{-12}{-3} \Rightarrow y = 4$$

Substituting the value of y in equation (2), we get

$x + 4 = 12 \Rightarrow x = 12 - 4 \Rightarrow x = 8$

Hence, the number $= 10 \times 8 + 4 = 80 + 4 = 84$

Question 17.

The result of dividing a number of two digits by the number with the digits reversed is $\frac{5}{6}$. If the difference of digits is 1, find the number.

Solution:

Let the digit at ten's place $= x$

and let the digit at unit place $= y$

Number $= 10 \times x + y \times 1 = 10x + y$

reversing the number $= 10 \times y + 1 \times x = 10y + x$

According to first condition of given problem,

$$\frac{10x+y}{10y+x} = \frac{5}{6} \Rightarrow 6(10x+y) = 5(10y+x)$$

$\Rightarrow 60x + 6y = 50y + 5x$

$\Rightarrow 60x + 6y - 50y - 5x = 0 \Rightarrow 55x - 44y = 0$

$\Rightarrow 11(5x - 4y) = 0 \Rightarrow 5x - 4y = 0$ ....(1)

According to second condition of given problem,

$y - x = 1 \Rightarrow -x + y = 1$ ....(2)

Multiplying equation (2) by 5, we get

$-5x + 5y = 5$ ....(3)

$5x - 4y = 0$

Adding, $y = 5$

Substituting the value of y in equation (2), we get

$-x + 5 = 1 \Rightarrow -x = 1 - 5 \Rightarrow -x = -4$

$\Rightarrow x = 4$

Hence, number $= 10x + y = 10 \times 4 + 5 = 40 + 5 = 45$

Question 18.

A number of three digits has the hundred digit 4 times the unit digit and the sum of three digits is 14. If the three digits are written in the reverse order, the value of the number is decreased by 594. Find the number.

Solution:

Let the digits at ten's place $= x$

Let the digits at unit place $= y$

then, digit at hundred place $= 4y$

number $= 100 \times 4y + 10 \times x + 1 \times y$

$= 400y + 10x + y = 401y + 10x = 10x + 401y$

reversing the number $= 100 \times y + 10 \times x + 1 \times 4y$

$= 100y + 10x + 4y = 104y + 10x = 10x + 104y$

According to first condition of the given problem,

$x + y + 4y = 14 \Rightarrow x + 5y = 14$ ....(1)

According to second condition of given problem,

$10x + 401y = 10x + 104y + 594$

$\Rightarrow 10x + 401y - 10x - 104y = 594$

$\Rightarrow 401y - 104y = 594 \Rightarrow 297y = 594$

$\Rightarrow y = \frac{594}{297} \Rightarrow y = 2$

Substituting the value of y in equation (1), we get

$x + 5 \times 2 = 14 \Rightarrow x + 10 = 14 \Rightarrow x = 14 - 10$

$\Rightarrow x = 4$

Hence, number $= 10x + 401y = 10 \times 4 + 401 \times 2$

$= 40 + 802 = 842$

Question 19.

Four years ago Marina was three times old as her daughter. Six years from now the mother will be twice as old as her daughter. Find their present ages.

Solution:

Let the present age of Marina = x years
and let the present age of Marina's daughter = y years.
Four years ago age of Marina's = $(x - 4)$ years
Four years ago age of Marina's daughter = $(y - 4)$ years.
According to first condition of given problem,

$x - 4 = 3(y - 4) \Rightarrow x - 4 = 3y - 12$

$\Rightarrow x - 3y = -12 + 4 \Rightarrow x - 3y = -8$ ....(1)

Age of Marina, six years from now = $(x + 6)$ years
Age of Marina's daughter, six years from now = $(y + 6)$ years.

According to second condition of given problem,

$x + 6 = 2(y + 6) \Rightarrow x + 6 = 2y + 12$

$\Rightarrow x - 2y = 12 - 6 \Rightarrow x - 2y = 6$ ....(2)

From equation (1) and (2), we get

$x - 3y = -8$ ....(1)

$x - 2y = 6$ ....(2)

$- \quad + \quad -$

Subtracting, $-y = -14$

$y = 14$

Substituting the value of y in equation (1), we get

$x - 3 \times 14 = -8 \Rightarrow x - 42 = -8 \Rightarrow x = -8 + 42$

$\Rightarrow x = 34$

Hence, Age of Marina = 34 years,
Age of Marina's Daughter = 14 years.

Question 20.

On selling a tea set at 5% loss and a lemon set at 15% gain, a shopkeeper gains Rs. 70. If he sells the tea set at 5% gain and lemon set at 10% gain, he gains Rs. 130. Find the cost price of the lemon set.

Solution:

Loss on tea set = 5%

and gain on lemon set = 15%

Let the C.P. of tea set = Rs. x

and C.P. of lemon set = Rs. y

According to the conditions,

$$\frac{y \times 15}{100} - \frac{x \times 5}{100} = 70$$

$$5y - 5x = 7000$$

$$3y - x = 1400 \quad \ldots(i)$$

and $$\frac{x \times 5}{100} + \frac{y \times 150}{100} = 130$$

$$5x + 150y = 13000$$

$$x + 2y = 2600 \quad \ldots(ii)$$

Adding (i) and (ii),

$$5y = 4000 \Rightarrow y = \frac{4000}{5} = 800$$

$\therefore$ Cost price of lemon set = Rs. 800

Question 21.

A person invested some money at 12% simple interest and some other amount at 10% simple interest. He received yearly interest of Rs, 1300. If he had interchanged the amounts, he would have received Rs. 40 more as yearly interest. How much did he invest at different rates ?

Solution:

Let amount invested at S.I. = Rs. x,

rate = 12% p.a.

and investment at S.I. = Rs. y,

rate = 10% p.a.

According to the condition,

$$\frac{x \times 12}{100} + \frac{y \times 10}{100} = 1300$$

$$\Rightarrow 12x + 10y = 130000 \Rightarrow 6x + 5y = 65000 \quad \ldots(i)$$

and $$\frac{10x}{100} + \frac{12y}{100} = 1340$$

$$\Rightarrow 10x + 12y = 134000 \Rightarrow 5x + 6y = 67000 \quad \ldots(ii)$$

Multiplying (i) by 6 and (ii) by 5, we get

$$36x + 30y = 390000$$
$$25x + 30y = 335000$$
$$- \quad - \quad -$$
$$11x = 55000$$

$$x = \frac{55000}{11} = 5000$$

Substituting the value of x in (i)

$$6 \times 5000 + 5y = 65000$$

$$30000 + 5y = 65000$$

$$5y = 65000 - 30000 = 35000$$

$$y = \frac{35000}{5} = 7000$$

$\therefore$ Investment at 12% = Rs. 5000

and investment at 10% = Rs. 7000

Question 22.

A shopkeeper sells a table at 8% profit and a chair at 10% discount, thereby getting Rs. 1008. If he had sold the table at 10% profit and chair at 8% discount, he would have got Rs. 20 more. Find the cost price of the table and the list price of the chair.

Solution:

Profit on table = 8%

and discount on chair = 10%

Let C.P. of table = Rs. x

and C.P. of chair = Rs. y

According to the condition,

$$\frac{x\times(100+8)}{100}+\frac{y\times(100-10)}{100}=1008$$

$$108x+90y=100800$$

$$6x+5y=5600 \quad \ldots(i)$$

Similarly,

$$\frac{x(100+10)}{100}+\frac{y(100-8)}{100}=1028$$

$$110x+92y=102800$$

$$55x+46y=51400 \quad \ldots(ii)$$

Multiplying (i) by 55 and (ii) by 6

$$\begin{array}{r} 330x+275y=308000 \\ 330x+276y=308400 \\ \hline -\quad\quad -\quad\quad\quad -\end{array}$$

Subtracts $\qquad -y \quad = -400$

$\Rightarrow y = 400$

Now $6x+5\times400=5600$

Substituting the value of y in (i)

$\Rightarrow 6x+2000=5600$

$\Rightarrow 6x=5600-2000$

$$\Rightarrow 6x=3600 \Rightarrow x=\frac{3600}{6}=600$$

$\therefore$ C.P. of table = Rs. 600

and C.P. of chair = Rs. 400

Question 23.

A and B have some money with them. A said to B, "if you give me Rs. 100, my money will become 75% of the money left with you." B said to A" instead if you give me Rs. 100, your money will become 40% of my money, How much money did A and B have originally ?

Solution:

Let A has money = x

and B has money = y

According to the condition,

$$x + 100 = (y - 100) \times \frac{75}{100}$$

$$\Rightarrow x + 100 = (y - 100) \times \frac{3}{4}$$

$$\Rightarrow 4x + 400 = 3y - 300 \Rightarrow 4x - 3y = -300 - 400$$

$$\Rightarrow 4x - 3y = -700 \quad ...(i)$$

Again, $x - 100 = \frac{40}{100}(y + 100)$

$$\Rightarrow x - 100 = \frac{2}{5}(y + 100) \Rightarrow 5x - 500 = 2y + 200$$

$$\Rightarrow 5x - 2y = 200 + 500 = 700 \quad ...(ii)$$

Multiplying (i) by 2 and (ii) by 3,

$$8x - 6y = -1400$$

$$15x - 6y = 2100$$

$$- \quad + \quad -$$

Subtracting, $-7x = -3500$

$$\Rightarrow x = \frac{-3500}{-7} = 500$$

Substituting the value of x in (i)

$$4 \times 500 - 3y = -700$$

$$\Rightarrow 2000 - 3y = -700$$

$$\Rightarrow -3y = -700 - 2000 = -2700 \Rightarrow y = \frac{-2700}{-3} = 900$$

Hence A has money = Rs. 500

and B has moeny = Rs. 900

Question 24.

The students of a class are made to stand in (complete) rows. If one student is extra in a row, there would be 2 rows less, and if one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Solution:

Let the number of students in one row $= x$

and let the number of rows $= y$

Then total number of students $= xy$

According to first condition of given problem,

$(x + 1)(y - 2) = xy \Rightarrow x(y - 2) + 1(y - 2) = xy$

$\Rightarrow xy - 2x + y - 2 = xy$

$\Rightarrow -2x + y - 2 = 0$

$\Rightarrow -2x + y = 2$ ....(1)

According to second condition of given problem,

$(x - 1)(y + 3) = xy$

$\Rightarrow x(y + 3) - 1(y + 3) = xy$

$\Rightarrow xy + 3x - y - 3 = xy$

$\Rightarrow 3x - y - 3 = 0$

$\Rightarrow 3x - y = 3$ ....(2)

From equation (1) and equation (2), we get

$$\begin{array}{rl} & -2x + y = 2 \\ & 3x - y = 3 \\ \hline \text{Adding,} & x = 5 \end{array}$$

Substituting the value of x in (1), we get

$-2 \times 5 + y = 2 \Rightarrow -10 + y = 2 \Rightarrow y = 2 + 10$

$\Rightarrow y = 12$

Hence, Number of students $= xy = 5 \times 12 = 60$

Question 25.

A jeweller has bars of 18-carat gold and 12- carat gold. How much of each must be melted together to obtain a bar of 16-carat gold weighing 120 grams ? (Pure gold is 24 carat)

Solution:

Let x gm of 18 carat gold and

Let y gm of 12 carat gold

Then according to first condition of given problem,

$x + y = 120$ ....(1)

Pure gold is 24 carat

Then purity of 18 carat gold $= \frac{18}{24} \times 100\%$

$= \frac{3}{4} \times 100 = 75\%$

Purity of 12 carat gold $= \frac{12}{24} \times 100\%$

$= \frac{1}{2} \times 100\% = 50\%$

Purity of 16 carat gold $= \frac{16}{24} \times 100\%$

$= \frac{2}{3} \times 100\% = \frac{200}{3}\%$

According to second condition of given problem,

$75x + 50y = \frac{200}{3} \times 120 \Rightarrow 75x + 50y = 200 \times 40$

$\Rightarrow 75x + 50y = 8000$ ....(2)

Multiplying equation (1) by 50, we get

$50x + 50y = 6000$

$75x + 50y = 8000$

$- \quad - \quad -$

Subtracting, $-25x = -2000$

$x = \frac{-2000}{-25} \Rightarrow x = 80$

Substituting the value of x in equation (1), we get

$80 + y = 120 \Rightarrow y = 120 - 80 \Rightarrow y = 40$

Hence, 80 gm of 18 carat gold

and 40 gm of 12 carat gold.

Question 26.

A and B together can do a piece of work in 15 days. If A's one day work is $1\frac{1}{2}$ times the one day's work of B, find in how many days can each do the work.

Solution:

Let A's one day work be $= x$

and B's one day work be $= y$

Then according to first condition of given problem,

$$x = \frac{3}{2}y \Rightarrow x - \frac{3}{2}y = 0 \Rightarrow \frac{2x-3y}{2} = 0$$

$$\Rightarrow 2x - 3y = 0 \quad \text{....(1)}$$

Also given that, In 15 days, A and B together can do a piece of work,

Then in 1 days A and B together can do $\frac{1}{15}$ piece of work,

Then, according to second condition of given problem,

$$x + y = \frac{1}{15} \Rightarrow 15(x+y) = 1$$

$$\Rightarrow 15x + 15y = 1 \quad \text{.....(2)}$$

Multiplying equation (1) by 5, we get

$$10x - 15y = 0$$

$$15x + 15y = 1$$

Adding, $\underline{25x \quad = 1}$

$$\Rightarrow x = \frac{1}{25}$$

Substituting the value of x in equation (1), we get

$$2 \times \frac{1}{25} - 3y = 0 \Rightarrow \frac{2}{25} = 3y$$

$$\Rightarrow y = \frac{2}{25 \times 3} = \frac{2}{75}$$

Hence, Man A do the work in days $= \frac{1}{x} = \frac{1}{\left(\frac{1}{25}\right)}$

$= 25$ days

Man B do the work in days, $= \frac{1}{y} = \frac{1}{\left(\frac{2}{75}\right)}$

$= \frac{75}{2} = 37\frac{1}{2}$ days

Question 27.

men and 5 women can do a piece of work in 4 days, while one man and one woman can finish it in 12 days. How long would it take for 1 man to do the work ?

Solution:

Let 1 man to do the work in x days and let 1 woman to do the work in y days,

then In 1 days 1 man do the work = $\frac{1}{x}$

In 1 days 1 woman do the work = $\frac{1}{y}$

In 1 days 2 man do the work = $\frac{2}{x}$

1 days 5 women do the work = $\frac{5}{y}$

According to first condition of given problem,

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4} \qquad \text{....(1)}$$

According to second condition of given problem,

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{12} \qquad \text{....(2)}$$

Multiplying equation (2) by 5, we get

$$\frac{5}{x}+\frac{5}{y}=\frac{5}{12} \qquad \text{....(3)}$$

$$\begin{array}{rrr} \frac{2}{x} & + \frac{5}{y} & = \frac{1}{4} \\ - & - & - \\ \hline \end{array}$$

Subtracting, $\frac{3}{x}=\frac{5}{12}-\frac{1}{4}$

$$\Rightarrow \frac{3}{x}=\frac{5-3}{12} \Rightarrow \frac{3}{x}=\frac{2}{12} \Rightarrow 3\times 12=2\times x$$

$$\Rightarrow 2\times x=3\times 12 \Rightarrow x=\frac{3\times 12}{2}=\frac{3\times 6}{1}$$

= 18 days

Hence, 1 man do the work in 18 days.

[If we find the value of y then substitute the value of x in equation (1) or (2)]

Question 28.

A train covered a certain distance at a uniform speed. If the train had been 30 km/hr faster, it would have taken 2 hours less than the scheduled time. If the train were slower by 15 km/hr, it would have taken 2 hours more than the scheduled time. Find the length of the journey.

Solution:

Let the due speed of the train be x km/hr and scheduled time be y hours.

Then (Distance) length of the journey = speed × time

$= xy$

According to first condition of given problem,

$(x+30)(y-2) = xy \Rightarrow x(y-2)+30(y-2) = xy$

$\Rightarrow xy - 2x + 30y - 60 = xy \Rightarrow -2(x-15y) = 60$

$$\Rightarrow 2(x-15y) = 60 \Rightarrow x - 15y = \frac{60}{-2}$$

$\Rightarrow x - 15y = -30$ ....(1)

According to second condition of given problem,

$(x-15)(y+2) = xy \Rightarrow x(y+2) - 15(y+2) = xy$

$\Rightarrow xy + 2x - 15y - 30 = xy \Rightarrow 2x - 15y - 30 = 0$

$\Rightarrow 2x - 15y = 30$ ....(2)

From equation (1) and equation (2), we get

$x - 15y = -30$

$2x - 15y = 30$

$- \quad + \quad -$

Subtracting, $-x = -60$

$\Rightarrow x = 60$

Substituting the value of x in equation (1), we get

$60 - 15y = -30 \Rightarrow -15y = -30 - 60$

$$\Rightarrow -15y = -90 \Rightarrow y = \frac{-90}{-15} \Rightarrow y = 6$$

Hence, length of the journey = 60 × 6 km = 360 km.

Question 29.

A boat takes 2 hours to go 40 km down the stream and it returns in 4 hours. Find the speed of the boat in still water and the speed of the stream.

Solution:

Let the speed of the boat in still water $= x$ km/hr

and the speed of the stream $= y$ km/hr.

speed of boat in the direction of the stream $= (x + y)$ km/hr

speed of boat in the opposite direction of the stream $= (x - y)$ km/hr

Distance = speed × time

According to first condition of given problem,

$40 = (x + y) \times 2 \Rightarrow 2(x + y) = 40$

$\Rightarrow x + y = \frac{40}{2} \Rightarrow x + y = 20$ ....(1)

According to second condition of given problem,

Distance = speed × time

$40 = (x - y) \times 4 \Rightarrow 4(x - y) = 40$

$\Rightarrow x - y = \frac{40}{4} \Rightarrow x - y = 10$ ....(2)

From equation (1) and (2), we get

$x + y = 20$ ....(1)

$x - y = 10$ ....(2)

Adding, $\overline{2x = 30}$

$\Rightarrow x = \frac{30}{2} \Rightarrow x = 15$

Substituting the value of x in equation (1), we get

$15 + y = 20 \Rightarrow y = 20 - 15 \Rightarrow y = 5$

Hence, speed of boat in still water = 15 km/hr

speed of the stream = 5 km/hr.

Question 30.

A boat sails a distance of 44 km in 4 hours with the current. It takes 4 hours 48 minutes longer to cover the same distance against the current. Find the speed of the boat in still water and the speed of the current.

Solution:

Let the speed of the boat = x km/hr

and the speed of the current = y = km/hr

Speed of the boat in the direction of current = $(x + y)$ km/hr

speed of boat in the opposite direction of current = $(x - y)$ km/hr

Distance = speed × time

According to first condition of given problem,

$44 = (x+y) \times 4 \Rightarrow 4(x+y) = 44$

$\Rightarrow x+y = \frac{44}{4} \Rightarrow x+y = 11$ ....(1)

According to second condition of given problem,

$44 = (x-y) \times \left(4\frac{48}{60} + 4\right) \quad \left(\because 4 \text{hours } 48 \text{min} = 4\frac{48}{60} \text{ hr}\right)$

$\Rightarrow 44 = (x-y) \times \left(4\frac{4}{5} + 4\right)$

$\Rightarrow 44 = (x-y) \times \left(\frac{24}{5} + 4\right)$

$\Rightarrow (x-y) \times \frac{44}{5} = 44 \Rightarrow x-y = \frac{44 \times 5}{44}$

$\Rightarrow x-y = \frac{11 \times 5}{11} \Rightarrow x-y = 5$ ....(2)

From equation (1) and equation (2), we get

$x+y = 11$ ....(1)

$x-y = 5$ ....(2)

Adding, $\overline{2x = 16}$

$\Rightarrow x = \frac{16}{2} \Rightarrow x = 8$

Substituting the value of x in equation (1), we get

$\Rightarrow 8+y = 11 \Rightarrow y = 11-8 \Rightarrow y = 3$

Hence, speed of the boat in still water = 8 km/hr

speed of the current = 3 km/hr

Question 31.

An aeroplane flies 1680 km with a head wind in 3.5 hours. On the return trip with same wind blowing, the plane takes 3 hours. Find the plane's air speed and the

wind speed.

Solution:

Let the speed of plane = x km/hr

and let the speed of wind = y km/hr

then speed of aeroplane in the direction of wind = $(x + y)$ km/hr

speed of aeroplane in the opposite direction of wind = $(x - y)$ km/hr

According to first condition of given problem,

Distance = speed × time

$1680 = (x - y) \times 3.5 \Rightarrow 3.5\,(x - y) = 1680$

$\Rightarrow x - y = \frac{1680}{3.5} \Rightarrow x - y = \frac{1680 \times 10}{35}$

$\Rightarrow x - y = \frac{1680 \times 2}{7} \Rightarrow x - y = 240 \times 2$

$\Rightarrow x - y = 480$ ....(1)

According to second condition of given problem,

Distance = speed × time

$1680 = (x + y) \times 3 \Rightarrow 3(x + y) = 1680$

$\Rightarrow x + y = \frac{1680}{3} \Rightarrow x + y = 560$ ....(2)

From equation (1) and (2), we get

$x - y = 480$

$x + y = 560$

Adding, $\underline{2x = 1040}$

$\Rightarrow x = \frac{1040}{2} = 520$

Substituting the value of x in equation (2), we get,

$520 + y = 560 \Rightarrow y = 560 - 520 \Rightarrow y = 40$

Hence, speed of aeroplane = 520 km/hr

and speed of wind = 40 km/hr.

Question 32.

A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When Bhawana takes food for 20 days, she has to pay Rs. 2600 as hostel charges; whereas when Divya takes food for 26 days, she pays Rs. 3020 as hostel charges. Find the fixed charges and the cost of food per day.

Solution:

Let fixed charges = Rs. x

and charges per day = Rs. y

According to the condition,

$$x + 20y = 2600 \quad (i)$$

$$x + 26y = 3020 \quad (ii)$$

$$- \quad - \quad -$$

Subtracting, $\quad -6y = -420$

$$y = \frac{-420}{-6} = 70$$

Substituting the value of y in (i)

$\Rightarrow x + 20 \times 70 = 2600 \Rightarrow x + 1400 = 2600$

$\Rightarrow x = 2600 - 1400 = 1200$

$\therefore$ Fixed charges = Rs. 1200

and daily charges = Rs. 70

Multiple Choice Questions

Choose the correct answer from the given four options (1 to 8):

Question 1.

Sum of digits of a two digit number is 8. If the number obtained by reversing the digits is 18 more than the original number, then the original number is

(a) 35

(b) 53

(c) 26

(d) 62

Solution:

Let unit digit of a number = x

Then tens digit = $8 - x$

$\therefore$ Number = $x + 10(8 - x) = x + 80 - 10x$

$= 80 - 9x$

By reversing the order of digit

Unit digit = $8 - x$ and tens digit = x

$\therefore$ Number = $8 - x + 10x = 8 + 9x$

$\therefore 8 + 9x = 80 - 9x + 18$

$\Rightarrow 9x + 9x = 80 + 18 - 8 = 90$

$$18x = 90 \Rightarrow x = \frac{90}{18} = 5$$

Question 2.

The sum of two natural numbers is 25 and their difference is 7. The numbers are

(a) 17 and 8

(b) 16 and 9

(c) 18 and 7

(d) 15 and 10

Solution:

Let x and y are two natural number

Then $x + y = 25$

$x - y = 7$

Adding, we get $2x = 32 \Rightarrow x = \frac{32}{2} = 16$

Subtracting, $2y = 18 \Rightarrow y = \frac{18}{2} = 9$

$\therefore$ Numbers are 16, 9 (b)

Question 3.

The sum of two natural numbers is 240 and their ratio is 3 : 5. Then the greater number is

(a) 180

(b) 160

(c) 150

(d) 90

Solution:

Let x and y the natural number, then

$x + y = 240$ ...(i)

and $\frac{x}{y} = \frac{3}{5} \Rightarrow 5x = 3y \Rightarrow x = \frac{3}{5}y$

Substituting the value of x in (i),

$\frac{3}{5}y + y = 240 \Rightarrow \frac{8}{5}y = 240$

$\Rightarrow y = \frac{240 \times 5}{8} = 150$

$\therefore$ Greater number = 150 (c)

Question 4.

The sum of the digits of a two digit number is 9. If 27 is added to it, the digits of the number get reversed. The number

(a) 27

(b) 72
(c) 63
(d) 36

Solution:

Sum of digits of a two digit number = 9

Let x be unit digit

Then ten's digit $= 9 - x$

$\therefore$ Number $= x + 10(9 - x)$

$= x + 90 - 10x = 90 - 9x$

By reversing the digits,

Unit digit $= 9 - x$ and tens digit $= x$

$\therefore$ Number $= 9 - x + 10x = 9 + 9x$

$\therefore 90 - 9x + 27 = 9 + 9x$

$\Rightarrow 117 - 9 = 9x + 9x \Rightarrow 18x = 108$

$$\Rightarrow x = \frac{108}{18} = 6$$

$\therefore$ Number $= 90 - 9x = 90 - 9 \times 6 = 90 - 54$

$= 36$ (d)

Question 5.

The sum of the digits of a two digit number is 12. If the number is decreased by 18, its digits get reversed. The number is

(a) 48
(b) 84
(c) 57
(d) 75

Solution:

Sum of digits of a two digits number = 12

Let unit digit = x

Then tens digit = $12 - x$

$\therefore$ Number = $x + 10(12 - x) = x + 120 - 10x$

$= 120 - 9x$

By reversing the digits,

Unit digit = $12 - x$ and tens digit = x

$\therefore$ Number = $12 - x + 10x = 12 + 9x$

$\therefore$ $120 - 9x - 18 = 12 + 9x$

$\Rightarrow$ $102 - 12 = 9x + 9x$

$\Rightarrow 18x = 90 \Rightarrow x = \frac{90}{18} = 5$

Number = $120 - 9x = 120 - 9 \times 5$

$= 120 - 45 = 75$ (d)

Question 6.

Aruna has only □1 and □2 coins with her. If the total number of coins that she has is 50 and the amount of the money with her is □75, then the number of □1 and □2 coins are, respectively

(a) 35 and 15

(b) 35 and 20

(c) 15 and 75

(d) 25 and 25

Solution:

Total number of coins = 50

and total amount of coins = ₹75

Let coins of ₹1 = x

and coins of ₹2 = $50 - x$

$\therefore$ $x \times 1 + (50 - x) \times 2 = 75$

$\Rightarrow$ $x + 100 - 2x = 75 \Rightarrow -x = 75 - 100 = -25$

$\Rightarrow$ $x = 25$

$\therefore$ Number of ₹1 coins = 25

and of ₹2 = 50 – 25 = 25 (d)

Question 7.

The age of a woman is four times the age of her daughter. Five years hence, the age of the woman will be three times the age of her daughter. The present age of the daughter is

(a) 40 years

(b) 20 years
(c) 15 years
(d) 10 years

Solution:

Let age of daughter = x years
Then age of woman = $4x$
5 years hence,
Age of daughter = $x + 5$
and age of woman = $4x + 5$
$\therefore\ 4x + 5 = 3(x + 5) \Rightarrow 4x + 5 = 3x + 15$
$\Rightarrow\ 4x - 3x = 15 - 5 \Rightarrow x = 10$
$\therefore$ Age of daughter = 10 years (d)

Question 8.

Father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present age in years of the son and the father are, respectively

(a) 4 and 24
(b) 5 and 30
(c) 6 and 36
(d) 3 and 24

Solution:

Let son's age = x years
Then age of his father = $6x$
4 years hence
Age of son = $x + 4$
and age of father = $6x + 4$
$\therefore\ 6x + 4 = 4\ (x + 4)$
$\Rightarrow\ 6x + 4 = 4x + 16 \Rightarrow 6x - 4x = 16 - 4$

$$\Rightarrow\ 2x = 12 \Rightarrow x = \frac{12}{2} = 6$$

Present age of son = 6 years
and age of father = $6 \times 6 = 36$ years (c)

Chapter Test

Question 1.

A 700 gm dry fruit packcosts Rs. 216. It contains some almonds and the rest cashew kernel. If almonds cost Rs. 288 'per kg and cashew kernel.cost Rs. 336 per kg, what are the quantities of the two dry fruits separately ?

Solution:

Cost of 700 gm of fruit = Rs. 216

Cost of almonds = Rs. 288 per kg

and cost of cashew = Rs. 336 per kg

Let weight of almond = x gm

Then weigh of cashew = $(700 - x)$ gm

According to the sum,

$$\frac{x \times 288}{1000} + \frac{(700-x) \times 366}{1000} = 216$$

$$288x + 235200 - 336x = 216000$$

$$\Rightarrow \quad 288x - 336x = 216000 - 235200$$

$$\Rightarrow \quad -48x = -19200$$

$$x = \frac{-19200}{-48} = 400$$

$\therefore$ weight of almond = 400 gm

and weight of cashew = 700 – 400

= 300 gm

Question 2.

Drawing pencils cost 80 paise each and coloured pencils cost Rs. 1.10 each. If altogether two dozen pencils cost Rs. 21.60, how many coloured pencils are there ?

Solution:

Let the number of drawing pencils $= x$

and let the number of coloured pencils $= y$

Then, according to first condition of given problem,

$x + y = 2 \times 12$ (2 dozen $= 2 \times 12$)

$\Rightarrow \quad x + y = 24$ ....(1)

According to second condition of given problem,

$$x \times \frac{80}{100} + y \times 1.10 = 21.60 \quad \left[\because 80\text{paise} = \text{Rs.}\frac{80}{100}\right]$$

$$\Rightarrow \quad \frac{80x}{100} + 1.10y = 21.60$$

$$\Rightarrow \quad \frac{80x + 1.10y \times 100}{100} = 21.60$$

$$\Rightarrow \quad 80x + 110y = 21.60 \times 100$$

$$\Rightarrow \quad 80x + 110y = 2160 \quad \Rightarrow \quad 10(8x + 11y) = 2160$$

$$\Rightarrow \quad 8x + 11y = \frac{2160}{10} \quad \Rightarrow \quad 8x + 11y = 216 \quad \text{....(2)}$$

Multiplying equation (1) by 8, we get

$8x + 8y = 192$ ....(3)

$8x + 11y = 216$ ....(4)

$- \quad - \quad -$

Subtracting, $-3y = -24$

$$\Rightarrow \quad y = \frac{-24}{-3} \quad \Rightarrow \quad y = 8$$

Substituting the value of y in equation (1), we get

$x + 8 = 24 \quad \Rightarrow \quad x = 24 - 8 \quad \Rightarrow \quad x = 16$

Hence, Number of coloured pencils $y = 8$

Question 3.

Shikha works in a factory. In one week she earned Rs. 390 for working 47 hours, of which 7 hours were overtime. The next week she earned Rs. 416 for working 50 hours, of which 8 hours were overtime. What is Shikha's hourly earning rate ?

Solution:

Let Shikha's earning be Rs. x per regular hour and Rs. y per hour overtime,

then according to first condition of given problem,

$40x + 7y = 390$ ....(1)

According to second condition of given problem,

$42x + 8y = 416$ ....(2)

Multiplying equation (1) by 8 and equation (2) by 7

$$320x + 56y = 3120$$
$$294x + 56y = 2912$$
$$- \quad - \quad -$$

Subtracting, $26x = 208$

$$\Rightarrow \quad x = \frac{208}{26} \quad \Rightarrow \quad x = 8$$

Substituting the value of x in equation (1), we get

$40 \times 8 + 7y = 390 \quad \Rightarrow \quad 320 + 7y = 390$

$\Rightarrow \quad 7y = 390 - 320 \quad \Rightarrow \quad 7y = 70 \quad \Rightarrow \quad y = 10$

Hence, Shika's earning be Rs. 8 per regular hours and Rs. 10 per hour overtime.

Question 4.

The sum of the digits of a two digit number is 7. If the digits are reversed, the new number increased by 3 equals 4 times the original number. Find the number.

Solution:

Let the digit at tens place $= x$

and let the digit at unit place $= y$

then number $= 10 \times x + 1 \times y = 10x + y$

Reversing the number $= 10 \times y + x + 1 = 10y + x$

According to first condition of given problem

$x + y = 7$ ....(1)

According to second condition of given problem,

$10y + x = 4(10x + y) - 3 \Rightarrow 10y + x = 40x + 4y - 3$

$\Rightarrow 10y + x - 40x - 4y = -3 \Rightarrow -39x + 6y = -3$

$\Rightarrow +3(-13x + 2y) = -3 \Rightarrow -3(13x - 2y) = -3$

$\Rightarrow 13x - 2y = \frac{-3}{-3} \Rightarrow 13x - 2y = 1$ ....(2)

Multiplying equation (1) by 2, we get

$2x + 2y = 14$ ....(3)

$13x - 2y = 1$ ....(4)

Adding, $\overline{15x = 15}$

$\Rightarrow x = \frac{15}{15} = 1$

Substituting the value of x in equation (1), we get

$1 + y = 7 \Rightarrow y = 7 - 1 \Rightarrow y = 6$

Hence, Number $= 10x + y = 10 \times 1 + 6$

$= 10 + 6 = 16$

Question 5.

Three years hence a man's age will be three times his son's age and 7 years ago he was seven times as old as his son. How old are they now ?

Solution:

Let the man's age now = x years

and his son's age now = y years

three years hence man's age = $(x + 3)$ years

three years hence his son's age = $(y + 3)$ years

7years ago man's age = $(x - 7)$ years

7 years ago his son's age = $(y - 7)$ years

According to first condition of given problem,

$(x+3) = 3(y+3) \Rightarrow x+3 = 3y+9$

$\Rightarrow x - 3y = 9 - 3 \Rightarrow x - 3y = 6$ ....(1)

According to second condition of given problem,

$(x-7) = 7(y-7) \Rightarrow x - 7 = 7y - 49$

$\Rightarrow x - 7y = -49 + 7 \Rightarrow x - 7y = -42$ ....(2)

From equation (1) and equation (2), we get

$x - 3y = 6$

$x - 7y = -42$

$- \quad + \quad +$

Subtracting, $+4y = 48$

$\Rightarrow y = \frac{48}{4} \Rightarrow y = 12$

Substituting the value of y in equation (1), we get

$x - 3 \times 12 = 6 \Rightarrow x - 36 = 6 \Rightarrow x = 6 + 36$

$\Rightarrow x = 42$

Hence, Man's age = 42 years

His son's age y years = 12 years

Question 6.

Rectangles are drawn on line segments of fixed lengths. When the breadths are 6 m and 5 m respectively the sum of the areas of the rectangles is 83 m². But if the breadths are 5 m and 4 m respectively the sum of the areas is 68 m². Find the sum of the areas of the squares drawn on the line segments.

Solution:

Let the length of first fixed line segment $= x$

and length of second line segment $= y$

then in first case sum of areas $= 83\ m^2$

and breadths are 6 m and 5 m respectively.

$\therefore 6x + 5y = 83$ ...(*i*)

In second case,

$5x + 4y = 68$ ...(*ii*)

Multiply (*i*) by 4 and (*ii*) by 5,

$24x + 20y = 332$

$25x + 20y = 340$

$- \quad - \quad -$

Subtracting, $-x = -8$

$\Rightarrow \quad x = 8$

Substituting the value of x in (*i*)

$6 \times 8 + 5y = 83$

$\Rightarrow \quad 48 + 5y = 83$

$\Rightarrow \quad 5y = 83 - 48 = 35$

$$y = \frac{35}{5} = 7$$

Hence first line segment = 8 m

and second line segment = 7 m

Now sum of areas of the square on these two line segments $= (8)^2 + (7)^2$

$= 64 + 49 = 113\ m^2$

Question 7.

If the length and the breadth of a room are increased by 1 metre each, the area is increased by 21 square metres. If the length is decreased by 1 metre and the breadth is increased by 2 metres, the area is increased by 14 square metres. Find the perimeter of the room.

Solution:

$= (x + 1)(y + 1)$ sq. metre

when the length is decreased by 1 metre, then new length $= (x - 1)$ m

when the breadth is increased by 2 metre, new breadth $= (y + 2)$ m

New Area = new length × New breadth

$= (x - 1)(y + 2)$ sq. metre

According to first condition of given problem,

$xy = (x + 1)(y + 1) - 21$

$\Rightarrow \quad xy = x(y + 1) + 1(y + 1) - 21$

$\Rightarrow \quad xy = xy + x + y + 1 - 21 \quad \Rightarrow \quad 0 = x + y - 20$

$\Rightarrow \quad 20 = x + y \quad \Rightarrow \quad x + y = 20 \qquad \ldots.(1)$

According to second condition of given problem,

$xy = (x - 1)(y + 2) - 14$

$\Rightarrow \quad xy = x(y + 2) - 1(y + 2) - 14$

$\Rightarrow \quad xy = xy + 2 - y - 2 - 14$

$\Rightarrow \quad 0 = 2x - y - 2 - 14 \quad \Rightarrow \quad 0 = 2x - y - 16$

$\Rightarrow \quad 16 = 2x - y \quad \Rightarrow \quad 2x - y = 16 \qquad \ldots.(2)$

From equation (1) and (2), we get

$x + y = 20 \qquad \ldots.(1)$

$2x - y = 16 \qquad \ldots.(2)$

Adding, $\underline{3x = 36}$

$\Rightarrow \quad x = \dfrac{36}{3} \quad x = 12$

Substituting the value of x in equation (1), we get

$12 + y = 20 \quad \Rightarrow \quad y = 20 - 12 \quad \Rightarrow \quad y = 8$

$\therefore$ Length of the room $= x$ m $= 12$ m

$\therefore$ Breadth of the room $= y$ m $= 8$ m

Hence perimeter of the room $= 2$ (length + breadth)

$= 2(12 + 8)$ metre $= 2 \times 20$ metre $= 40$ metre

Question 8.

The lenghts (in metres) of the sides of a triangle are $2x + \frac{y}{2}$, $\frac{5x}{3} + y + \frac{1}{2}$ and $\frac{2}{3}x + 2y + \frac{5}{2}$. If the triangle is equilateral, find its perimeter.

Solution:

Length of the sides of an equilateral triangle are

$$2x + \frac{y}{2}, \frac{5x}{3} + y + \frac{1}{2} \text{ and } \frac{2}{3}x + 2y + \frac{5}{2}$$

$$\therefore \quad \text{Perimeter} = 2x + \frac{y}{2} + \frac{5x}{3} + y + \frac{1}{2} + \frac{2}{3}x + 2y + \frac{5}{2}$$

$$= 2x + \frac{5x}{3} + \frac{2}{3}x + \frac{y}{2} + y + 2y + \frac{1}{2} + \frac{5}{2}$$

$$= \frac{6+5+2}{3}x + \frac{y+2y+4y}{2} + \frac{1+5}{2}$$

$$= \frac{13}{3}x + \frac{7}{2}y + 3$$

we know that sides of an equilateral are equal

$$\therefore \quad 2x + \frac{y}{2} = \frac{5x}{3} + y + \frac{1}{2}$$

$$\Rightarrow \quad 2x - \frac{5x}{3} + \frac{y}{2} - y = \frac{1}{2}$$

$$\Rightarrow \quad \frac{6x-5x}{3} + \frac{y-2y}{2} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{x}{3} - \frac{y}{2} = \frac{1}{2}$$

$$\Rightarrow \quad 2x - 3y = 3 \qquad \ldots \text{(i)}$$

$$\frac{5x}{3} + 0y + \frac{1}{2} = \frac{2}{3}x + 2y + \frac{5}{2}$$

$$\Rightarrow \quad \frac{5x}{3} - \frac{2}{3}x + y - 2y = \frac{5}{2} - \frac{1}{2}$$

$$\Rightarrow \quad \frac{5-2}{3}x - y = \frac{5-1}{2}$$

$\Rightarrow \quad x - y = 2 \qquad \ldots$ (ii)

from (ii), $x = 2 + y$

Substituting the value of x in (i)

$2(2 + y) - 3y = 3$

$\Rightarrow \quad 4 + 2y - 3y = 3$

$\Rightarrow \quad -y = 3 - 4 \Rightarrow \quad -y = -1$

$\Rightarrow \quad y = 1$

$\therefore \quad x = 2 + y = 2 + 1 = 3$

$\therefore \quad x = 3, y = 1$

$\therefore \quad$ Perimeter $= \frac{13}{3}x + \frac{7}{2}y + 3$

$$= \frac{13}{3} \times 3 + \frac{7}{2} \times 1 + 3 = 13 + \frac{7}{2} + 3 = 16 + 3\frac{1}{2} = 19\ \frac{1}{2} = 19.5 \text{ metres}$$

Question 9.

On Diwali eve, two candles, one of which is 3 cm longer than the other are lighted. The longer one is lighted at 530 p.m. and the shorter at 7 p.m. At 930 p.m. they both are of the same length. The longer one burns out at 1130 p.m. and the shorter one at 11 p.m. How long was each candle originally ?

Solution:

Let the longer candle shorten at the rate of x cm/hr in burning case and the smaller candle shorter at the rate of y cm/hr.

In burning case the longer candle burns out completely in 6 hours and that the smaller candle in 4 hours,

$\therefore$ Their lengths are $6x$ cm and $4y$ cm respectively

According to first condition of given problem,

$6x = 4y + 3 \Rightarrow 6x - 4y = 3$ ....(1)

At 9.30 p.m the length of longer candle $= (6x - 4x)$ cm $= 2x$ cm

At 9.30 p.m the length of smaller candle $= (4y - \frac{5}{2}y)$ cm $= \frac{8y - 5y}{2}$ cm $= \frac{3y}{2}$ cm

As According to second condition of given problem,

$\therefore \quad 2x = \frac{3}{2}y$

(Both candles have same length at 9.30 p.m.)

$\Rightarrow \quad 4x = 3y \Rightarrow 4x - 3y = 0$ ....(2)

Multiplying equation (1) by 3 and equation (2) by 4

$18x - 12y = 9$ ....(3)

$16x - 12y = 0$ ....(4)

$- \quad + \quad -$

Subtracting, $2x = 9$

$\therefore \quad x = \frac{9}{2} = 4.5$ cm/hr

Substituting the value of x in equation (2), we get

$4 \times 4.5 - 3y = 0 \Rightarrow 18 - 3y = 0 \Rightarrow 18 = 3y \Rightarrow y = \frac{18}{3} \quad \therefore \quad y = 6$ cm/hr.

Hence, lengths of longer candle $= 6x = 6 \times 4.5$ cm $= 27$ cm

length of smaller candle $= 4y$ cm $= 4 \times 6$ cm $= 24$ cm