

Chapter 8 Indices

7. Indices

EXERCISE - 7

Solution - (1)

$$(i) \left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$\rightarrow \left(\frac{3^4}{2^4}\right)^{-\frac{3}{4}}$$

$$\rightarrow \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}}$$

$$\rightarrow \left(\frac{3}{2}\right)^{4 \times -\frac{3}{4}}$$

$$\rightarrow \left(\frac{3}{2}\right)^{-3}$$

$$\rightarrow \left(\frac{2}{3}\right)^3$$

$$\rightarrow \frac{2^3}{3^3}$$

$$\rightarrow \frac{2 \times 2 \times 2}{3 \times 3 \times 3}$$

$$\rightarrow \frac{8}{27}$$

$$(ii) \left(1 \frac{61}{64}\right)^{-2/3}$$

$$\Rightarrow \left(\frac{125}{64}\right)^{-2/3}$$

$$\Rightarrow \left(\frac{5^3}{4^3}\right)^{-2/3}$$

$$\Rightarrow \left[\left(\frac{5}{4}\right)^3\right]^{-2/3}$$

$$\Rightarrow \left(\frac{5}{4}\right)^{3 \times -\frac{2}{3}}$$

$$\Rightarrow \left(\frac{5}{4}\right)^{-2}$$

$$\Rightarrow \left(\frac{4}{5}\right)^2$$

$$\Rightarrow \frac{4^2}{5^2}$$

$$\Rightarrow \frac{16}{25}$$

Solution-2

$$(i) (2a^{-3}b^2)^3$$

$$\Rightarrow 2^3 a^{-3 \times 3} b^{2 \times 3}$$

$$\Rightarrow 8 a^{-9} b^6$$

$$(ii) \quad \frac{a^{-1} + b^{-1}}{(ab)^{-1}}$$

$$\Rightarrow \frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{ab}}$$

$$\Rightarrow \frac{\frac{b+a}{ab}}{\frac{1}{ab}}$$

$$\Rightarrow \frac{b+a}{ab} \times \frac{ab}{1}$$

$$\Rightarrow a+b$$

Solution-3:

$$(i) \quad \frac{x^{-1}y^{-1}}{x^{-1}+y^{-1}}$$

$$\Rightarrow \frac{(xy)^{-1}}{\frac{1}{x} + \frac{1}{y}}$$

$$\Rightarrow \frac{\frac{1}{xy}}{\frac{y+x}{xy}}$$

$$\Rightarrow \frac{1}{xy}$$

$$(ii) \quad \frac{(4 \times 10^7) (6 \times 10^{-5})}{8 \times 10^{10}}$$

$$\rightarrow \frac{4 \times 10^7 \times 6 \times 10^{-5}}{8 \times 10^{10}}$$

$$\rightarrow \frac{3 \times 10^{7-5}}{10^{10}}$$

$$\rightarrow \frac{3 \times 10^2}{10^{10}}$$

$$\Rightarrow \frac{3}{10^{10-2}}$$

$$\Rightarrow \frac{3}{10^8}$$

solution-4 :

$$(i) \quad \frac{3a}{b^{-1}} + \frac{2b}{a^{-1}}$$

$$\rightarrow \frac{3a}{\left(\frac{1}{b}\right)} + \frac{2b}{\left(\frac{1}{a}\right)}$$

$$\Rightarrow 3ab + 2ab$$

$$\Rightarrow 5ab$$

$$(ii) \quad 5^0 \times 4^{-1} + 8^{1/3}$$

$$(a^0 = 1)$$

$$\rightarrow 1 \times \frac{1}{4} + (2^3)^{1/3}$$

$$\rightarrow \frac{1}{4} + 2$$

$$\rightarrow \frac{1+8}{4}$$

$$\rightarrow \frac{9}{4}$$

Solution - 5

$$(i) \quad \left(\frac{8}{125}\right)^{-1/3}$$

$$\Rightarrow \left(\frac{2^3}{5^3}\right)^{-1/3}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{3 \times -1/3}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{-1}$$

$$\Rightarrow \frac{5}{2}$$

$$(ii) \quad (0.027)^{-1/3}$$

$$\Rightarrow \left(\frac{27}{1000}\right)^{-1/3}$$

$$\Rightarrow \left(\frac{3^3}{10^3}\right)^{-1/3}$$

$$\Rightarrow \left(\frac{3}{10}\right)^{2x^{-1/3}}$$

$$\Rightarrow \left(\frac{3}{10}\right)^{-1}$$

$$\Rightarrow \frac{10}{3}$$

Solution-6 :

$$(i) \left(-\frac{1}{27}\right)^{-2/3}$$

$$\Rightarrow -\left(\frac{1}{3^3}\right)^{-2/3}$$

$$\Rightarrow -\frac{1}{(3^3)^{-2/3}}$$

$$\Rightarrow -\frac{1}{3^{2 \times -2/3}}$$

$$\Rightarrow -\frac{1}{3^{-2}}$$

$$\Rightarrow -3^2$$

$$\Rightarrow -9 //$$

$$(ii) (64)^{-2/3} \div 9^{-3/2}$$

$$\Rightarrow (4^3)^{-2/3} \div (9^2)^{-3/2}$$

$$\Rightarrow 4^{2 \times -2/3} \div 3^{2 \times -3/2}$$

$$\Rightarrow 4^{-2} \div 3^{-3}$$

$$\Rightarrow \frac{4^{-2}}{3^{-3}}$$

$$\Rightarrow \frac{\frac{1}{4^2}}{\frac{1}{3^3}}$$

$$\Rightarrow \frac{3^3}{4^2}$$

$$\Rightarrow \frac{27}{16}$$

Solution - 7

$$(i) \frac{(27)^{2n/3} \times 8^{-n/6}}{(18)^{-n/2}}$$

$$\Rightarrow \frac{(3^3)^{\frac{2n}{3}} \times (2^3)^{-n/6}}{(2 \times 9)^{-n/2}}$$

$$\Rightarrow \frac{3^{2 \times \frac{2n}{3}} \times 2^{3 \times \frac{-n}{6}}}{2^{-n/2} \times 3^{2 \times \frac{-n}{2}}}$$

$$\Rightarrow \frac{3^{2n} \times 2^{-n/2}}{2^{-n/2} \times 3^{-n}}$$

$$\Rightarrow \frac{3^{2n}}{2^{-n/2}} \times \frac{1}{2^{n/2}} \times 3^n$$

$$\Rightarrow \frac{3^{2n+n}}{2^{-n/2} \cdot 2^{n/2}}$$

$$\Rightarrow \frac{3^{3n}}{2^{-n/2} \cdot 2^{n/2}}$$

$$\Rightarrow \frac{3^{3n} \times \cancel{2^{n/2}}}{\cancel{2^{n/2}}}$$

$$\Rightarrow 3^{3n} //$$

$$(ii) \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}}$$

$$\Rightarrow \frac{5^1 \cdot (5^2)^{n+1} - 5^2 \cdot 5^{2n}}{5^1 \cdot 5^{2n+3} - (5^2)^{n+1}}$$

$$\Rightarrow \frac{5^{1+2n+2} - 5^{2+2n}}{5^{1+2n+3} - 5^{2n+2}}$$

$$\Rightarrow \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$$

$$\rightarrow \frac{5^{2n+3} - 5^2 \cdot 5^{2n}}{5^{2n} \cdot 5^4 - 5^2 \cdot 5^{2n}}$$

$$\rightarrow \frac{5^{\cancel{2n}} [5^3 - 5^2]}{5^{\cancel{2n}} [5^4 - 5^2]}$$

$$\rightarrow \frac{125 - 25}{625 - 25}$$

$$\rightarrow \frac{100}{600}$$

$$\rightarrow \frac{1}{6}$$

Solution-8

$$(i) \left(8^{-4/3} \div 2^{-2} \right)^{1/2}$$

$$\rightarrow \left((2^3)^{-4/3} \div 2^{-2} \right)^{1/2}$$

$$\rightarrow \left(\frac{2^{-4}}{2^{-2}} \right)^{1/2}$$

$$\rightarrow (2^{-4+2})^{1/2}$$

$$\rightarrow (2^{-2})^{1/2}$$

$$\rightarrow 2^{-1}$$

$$\rightarrow \frac{1}{2} //$$

$$(ii) \left(\frac{27}{8}\right)^{2/3} - \left(\frac{1}{4}\right)^{-2} + 5^0$$

$$\Rightarrow \left(\frac{3^3}{2^3}\right)^{2/3} - \left(\frac{1}{2^2}\right)^{-2} + 1$$

$$\Rightarrow \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - \frac{1}{2^{2 \times -2}} + 1$$

$$\Rightarrow \left(\frac{3}{2}\right)^2 - \frac{1}{2^{-4}} + 1$$

$$\Rightarrow \frac{9}{4} - 2^4 + 1$$

$$\Rightarrow \frac{9}{4} - 16 + 1$$

$$\Rightarrow \frac{9}{4} - 15$$

$$\Rightarrow \frac{9 - 60}{4}$$

$$\Rightarrow \frac{-51}{4}$$

Solution - 9

$$(i) (3x^2)^{-3} \times (x^9)^{2/3}$$

$$\Rightarrow \frac{1}{(3x^2)^3} \times x^{9 \times 2/3}$$

$$\Rightarrow \frac{1}{3^3 \cdot x^{2 \times 3}} \times x^{3 \times 2}$$

$$\Rightarrow \frac{1}{27 \cdot x^6} \times x^6$$

$$\Rightarrow \frac{1}{27}$$

$$(ii) (8x^4)^{1/3} \div x^{1/3}$$

$$\rightarrow (2^3 \cdot x^4)^{1/3} \div x^{1/3}$$

$$\rightarrow \frac{2^{3 \cdot \frac{1}{3}} \cdot x^{4 \cdot \frac{1}{3}}}{x^{1/3}}$$

$$\Rightarrow \frac{2 \cdot x^{4/3}}{x^{1/3}}$$

$$\Rightarrow 2 \cdot x^{4 \cdot \frac{1}{3}} - x^{1/3}$$

$$\Rightarrow x^{1/3} [2x^4 - 1] //$$

Solution-10

$$(i) (3^2)^0 + 3^{-4} \times 3^6 + \left(\frac{1}{3}\right)^{-2}$$

$$= 3^0 + 3^{-4+6} + \frac{1}{3^{-2}}$$

$$\rightarrow 1 + 3^2 + 3^2$$

$$\rightarrow 1 + \frac{1}{3^2} + 3^2$$

$$\rightarrow 1 + 9 + 9$$

$$\rightarrow 19 //$$

$$(ii) 9^{5/2} - 3 \cdot (5)^0 - \left(\frac{1}{81}\right)^{-1/2}$$

$$\rightarrow 9^{5/2} - 3(1) - \left(\frac{1}{9^2}\right)^{-1/2}$$

$$\rightarrow 9^{5/2} - 3 - \frac{1}{9^{2 \times -1/2}}$$

$$\rightarrow 9^{5/2} - 3 - \frac{1}{9^{-1}}$$

$$\rightarrow 3^5 - 3 - 9$$

$$\rightarrow 3^5 - 3 - 3^2$$

$$\rightarrow 3(3^4 - 1 - 3)$$

$$\rightarrow 3(81 - 1 - 3)$$

$$\rightarrow 3(77)$$

$$\rightarrow 231$$

Solution - 11

$$(i) \quad 16^{3/4} + 2 \left(\frac{1}{2}\right)^{-1} \cdot 3^0$$

$$\Rightarrow (2^4)^{3/4} + 2 \left(\frac{1}{2^{-1}}\right) \cdot 1$$

$$\Rightarrow 2^{4 \times \frac{3}{4}} + 2 \cdot 2$$

$$\Rightarrow 2^3 + 4$$

$$\Rightarrow 8 + 4$$

$$\Rightarrow 12$$

$$(ii) \quad (81)^{3/4} - \left(\frac{1}{32}\right)^{-2/5} + (8)^{1/3} \left(\frac{1}{2}\right)^{-1} \cdot (2)^0$$

$$\Rightarrow (3^4)^{3/4} - \left(\frac{1}{2^5}\right)^{-2/5} + (2^3)^{1/3} \left(\frac{1}{2^{-1}}\right) \cdot 1$$

$$\Rightarrow 3^3 - \frac{1}{2^{8 \times -2/5}} + 2 \left(\frac{1}{2}\right) \cdot 1$$

$$\Rightarrow 3^3 - \frac{1}{2^{-2}} + 2(2)$$

$$\Rightarrow 27 - 2^2 + 4$$

$$\Rightarrow 27 - 4 + 4$$

$$\Rightarrow 27$$

Solution-12

$$(i) \left(\frac{64}{125}\right)^{-2/3} \div \frac{1}{\left(\frac{256}{625}\right)^{1/4}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$$

$$\rightarrow \left(\frac{4^3}{5^3}\right)^{-2/3} \div \frac{1}{\left(\frac{4^4}{5^4}\right)^{1/4}} + 1$$

$$\rightarrow \left(\frac{4}{5}\right)^{3 \times -2/3} \div \frac{1}{\left(\frac{4}{5}\right)^{4 \times 1/4}} + 1$$

$$\rightarrow \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)} + 1$$

$$\rightarrow \frac{\left(\frac{5}{4}\right)^2}{\left(\frac{4}{5}\right)} + 1$$

$$\rightarrow \left(\frac{5}{4}\right)^2 \times \left(\frac{5}{4}\right) + 1$$

$$\rightarrow \frac{5}{4} + 1$$

$$\rightarrow \frac{5+4}{4}$$

$$\rightarrow \frac{9}{4}$$

$$(ii) \frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 22 \times 5^n}$$

$$\Rightarrow \frac{5^n \cdot 5^3 - 6 \times 5^n \cdot 5}{9 \times 5^n - 22 \times 5^n}$$

$$\Rightarrow \frac{5^n [5^3 - 6 \times 5]}{5^n [9 - 22]}$$

$$\Rightarrow \frac{125 - 30}{-13}$$

$$\Rightarrow \frac{95}{-13}$$

$$\Rightarrow -7.3077$$

Solution-13 :

$$(i) \left[(64)^{2/3} \cdot 2^{-2} \div 8^0 \right]^{-1/2}$$

$$\Rightarrow \left((4^3)^{2/3} \cdot \frac{1}{2^2} \div 1 \right)^{-1/2}$$

$$\Rightarrow \left(\frac{4^2}{2^2} \right)^{-1/2}$$

$$\Rightarrow \left(\frac{4}{2} \right)^{-1}$$

$$\Rightarrow 2^{-1}$$

$$\Rightarrow \frac{1}{2} //$$

$$(ii) \quad 3^n \times 9^{n+1} \div 3^{n-1} \times 9^{n-1}$$

$$\rightarrow 3^n \times 3^{2(n+1)} \div 3^{n-1} \times 3^{2(n-1)}$$

$$\Rightarrow 3^n \times 3^{2n+2} \div 3^{n-1} \times 3^{2n-2}$$

$$\Rightarrow \frac{3^{n+2n+2}}{3^{n-1+2n-2}}$$

$$\Rightarrow \frac{3^{3n+2}}{3^{3n-3}}$$

$$\Rightarrow \frac{\cancel{3^{3n}} \cdot 3^2}{\cancel{3^{3n}} \cdot 3^{-3}}$$

$$\Rightarrow 3^2 \times 3^{+3}$$

$$\Rightarrow 3^{2+3}$$

$$\Rightarrow 3^5$$

$$\Rightarrow 243$$

Solution- 14

$$(i) \quad \frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - \left(\frac{1}{2}\right)^{-2}$$

$$\Rightarrow \frac{(2^2)^{1/2} \times (4^4)^{1/4}}{(4^3)^{1/3}} - \frac{1}{2^{-2}}$$

$$\Rightarrow \frac{2 \times 4}{4} - 2^2$$

$$\Rightarrow 2 - 4$$

$$\Rightarrow -2.$$

$$(ii) \frac{3^{-6/7} \times 4^{-3/7} \times 9^{3/7} \times 2^{4/7}}{2^2 + 2^0 + 2^{-2}}$$

$$\Rightarrow \frac{3^{-6/7} \times 3^{2 \cdot \frac{3}{7}} \times 2^{2 \times \frac{3}{7}} \times 2^{6/7}}{4 + 1 + \frac{1}{2^2}}$$

$$\Rightarrow \frac{3^{-6/7} \times 3^{6/7} \times 2^{-6/7} \times 2^{6/7}}{4 + 1 + \frac{1}{4}}$$

$$\Rightarrow \frac{3^{-\frac{6}{7} + \frac{6}{7}} \times 2^{-\frac{6}{7} + \frac{6}{7}}}{\frac{16 + 4 + 1}{4}}$$

$$\Rightarrow \frac{3^0 \times 2^0}{\left(\frac{21}{4}\right)}$$

$$\Rightarrow \frac{1}{\left(\frac{21}{4}\right)}$$

$$\Rightarrow \frac{4}{21} //$$

Solution - 15

$$(i) \frac{(32)^{2/5} \times (4)^{-1/2} \times (8)^{1/3}}{2^{-2} \div (64)^{-1/3}}$$

$$\Rightarrow \frac{(2^{\cancel{5}})^{-2/\cancel{5}} \times (2^{\cancel{2}})^{-1/\cancel{2}} \times (2^{\cancel{3}})^{1/\cancel{3}}}{\frac{1}{2^2} \div (4^{\cancel{3}})^{-1/\cancel{3}}}$$

$$\Rightarrow \frac{2^{-2} \times 2^{-1} \times 2^1}{\frac{1}{2^2} \div (4^{-1})}$$

$$\Rightarrow \frac{2^{-1-2+1}}{\frac{(\frac{1}{2^2})}{(\frac{1}{2^2})}}$$

$$\Rightarrow 2^{-2}$$

$$\Rightarrow \frac{1}{2^2}$$

$$\Rightarrow \frac{1}{4}$$

$$(ii) \frac{5^{2(x+6)} \times 25^{-7+2x}}{(125)^{2x}}$$

$$\Rightarrow \frac{5^{2x+12} \times 5^2(-7+2x)}{(5^3)^{2x}}$$

$$\Rightarrow \frac{5^{2x+12} \times 5^{-14+4x}}{5^{6x}}$$

$$\Rightarrow \frac{5^{2x+12-14+4x}}{5^{6x}}$$

$$\Rightarrow \frac{5^{6x-2}}{5^{6x}}$$

$$\Rightarrow \frac{\cancel{5^{6x}} \cdot 5^{-2}}{\cancel{5^{6x}}}$$

$$\Rightarrow 5^{-2}$$

$$\Rightarrow \frac{1}{5^2}$$

$$\Rightarrow \frac{1}{25}$$

Solution- 16

$$(i) \quad \frac{7^{2n+3} - 49^{n+2}}{((343)^{n+1})^{2/3}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2(n+2)}}{(7^{3(n+1)})^{2/3}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2n+4}}{7^{2(n+1)}}$$

$$\Rightarrow \frac{7^{2n+3} - 7^{2n+4}}{7^{2n+2}}$$

$$\Rightarrow \frac{7^{2n} \cdot 7^3 - 7^{2n} \cdot 7^4}{7^{2n} \cdot 7^2}$$

$$\Rightarrow \frac{7^{2n} [7^3 - 7^4]}{7^{2n} \cdot 7^2}$$

$$\Rightarrow \frac{243 - 2401}{49}$$

$$\Rightarrow \frac{-2058}{49}$$

$$\Rightarrow -42 //$$

$$(ii) (27)^{4/3} + (32)^{0.8} + (0.8)^{-1}$$

$$\Rightarrow (3^3)^{4/3} + (2^5)^{4/5} + \left(\frac{8}{10}\right)^{-1}$$

$$\Rightarrow 3^4 + 2^{5 \times \frac{4}{5}} + \left(\frac{4}{5}\right)^{-1}$$

$$\Rightarrow 3^4 + 2^4 + \frac{5}{4}$$

$$\Rightarrow 81 + 16 + \frac{5}{4}$$

$$\Rightarrow 97 + \frac{5}{4}$$

$$\Rightarrow \frac{388 + 5}{4}$$

$$\Rightarrow \frac{393}{4}$$

Solution-17

$$(i) (\sqrt{32} - \sqrt{5})^{1/3} \cdot (\sqrt{32} + \sqrt{5})^{1/3}$$

$$\Rightarrow \left[(\sqrt{2^5} - \sqrt{5}) \cdot (\sqrt{2^5} + \sqrt{5}) \right]^{1/3}$$

$$\Rightarrow \left[(\sqrt{2^5})^2 - (\sqrt{5})^2 \right]^{1/3}$$

$$\Rightarrow (2^5 - 5)^{1/3}$$

$$\rightarrow (32-5)^{1/3}$$

$$\rightarrow (27)^{1/3}$$

$$\rightarrow (3^3)^{1/3}$$

$$\rightarrow \underline{3}$$

$$(ii) (x^{1/3} - x^{-1/3}) (x^{2/3} + 1 + x^{-2/3})$$

$$\Rightarrow \left(x^{1/3} - \frac{1}{x^{1/3}}\right) \left(x^{2/3} + 1 + \frac{1}{x^{2/3}}\right)$$

\therefore It is in the form of

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3$$

$$\therefore \text{Here } a = x^{1/3} ; b = \frac{1}{x^{1/3}}$$

$$\therefore (x^{1/3})^3 - \left(\frac{1}{x^{1/3}}\right)^3$$

$$\Rightarrow x^{4/3} - \frac{1}{x^{2/3}}$$

$$\Rightarrow x - \frac{1}{x} //$$

Solution - 18 :

$$(i) \left(\frac{x^m}{x^n}\right)^l \cdot \left(\frac{x^n}{x^l}\right)^m \cdot \left(\frac{x^l}{x^m}\right)^n$$

$$\rightarrow (x^{m-n})^l \cdot (x^{n-l})^m \cdot (x^{l-m})^n$$

$$\rightarrow x^{ml-nl} \cdot x^{mn-ml} \cdot x^{nl-nm}$$

$$\rightarrow x^{ml-nl+mn-ml+nl-nm}$$

$$\rightarrow x^0$$

$$\rightarrow 1$$

$$(ii) \left(\frac{x^{a+b}}{x^c}\right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^a}\right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^b}\right)^{c-a}$$

$$\rightarrow \frac{x^{(a+b)(a-b)}}{x^{c(a-b)}} \cdot \frac{x^{(b+c)(b-c)}}{x^{a(b-c)}} \cdot \frac{x^{(c+a)(c-a)}}{x^{b(c-a)}}$$

$$\rightarrow \frac{x^{a^2-b^2}}{x^{ac-bc}} \cdot \frac{x^{b^2-c^2}}{x^{ab-ac}} \cdot \frac{x^{c^2-a^2}}{x^{bc-ab}}$$

$$\rightarrow \frac{x^{a^2-b^2+b^2-c^2+c^2-a^2}}{x^{ac-bc+ab-ac+bc-ab}}$$

$$\rightarrow \frac{x^0}{x^0}$$

$$\rightarrow 1$$

Solution-19:

$$(i) \quad \sqrt[lm]{\frac{x^l}{x^m}} \cdot \sqrt[mn]{\frac{x^m}{x^n}} \cdot \sqrt[nl]{\frac{x^n}{x^l}}$$

$$\Rightarrow \sqrt[lm]{x^{l-m}} \cdot \sqrt[mn]{x^{m-n}} \cdot \sqrt[nl]{x^{n-l}}$$

$$\Rightarrow (x^{l-m})^{\frac{1}{lm}} \cdot (x^{m-n})^{\frac{1}{nm}} \cdot (x^{n-l})^{\frac{1}{nl}}$$

$$\Rightarrow x^{\frac{l-m}{lm}} \cdot x^{\frac{m-n}{nm}} \cdot x^{\frac{n-l}{nl}}$$

$$\Rightarrow x^{\frac{l-m}{lm} + \frac{m-n}{nm} + \frac{n-l}{nl}}$$

$$\Rightarrow x^{\frac{n(l-m) + l(m-n) + m(n-l)}{lmn}}$$

$$\Rightarrow x^{\frac{nl - nm + lm - ln + mn - lm}{lmn}}$$

$$\Rightarrow x^0$$

$$\Rightarrow 1$$

$$(ii) \quad \left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2}$$

$$\Rightarrow x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ca+a^2)}$$

$$\Rightarrow x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3}$$

$$\Rightarrow x^{a^3 - b^3 + b^3 - c^3 + c^3 - a^3}$$

$$\Rightarrow x^0$$

$$\Rightarrow 1$$

$$(iii) \left(\frac{x^a}{x-b}\right)^{a^2-ab+b^2} \cdot \left(\frac{x^b}{x-c}\right)^{b^2-bc+c^2} \cdot \left(\frac{x^c}{x-a}\right)^{c^2-ca+a^2}$$

$$\Rightarrow x^{(a-b)(a^2-ab+b^2)} \cdot x^{(b-c)(b^2-bc+c^2)} \cdot x^{(c-a)(c^2-ca+a^2)}$$

$$\Rightarrow x^{(a+b)(a^2-ab+b^2)} \cdot x^{(b+c)(b^2-bc+c^2)} \cdot x^{(c+a)(c^2-ca+a^2)}$$

$$\Rightarrow x^{a^3+b^3} \cdot x^{b^3+c^3} \cdot x^{c^3+a^3}$$

$$\Rightarrow x^{a^3+b^3+b^3+c^3+c^3+a^3}$$

$$\Rightarrow x^{2a^3+2b^3+2c^3}$$

$$\Rightarrow x^{2(a^3+b^3+c^3)}$$

Solution - 20:

$$(i) (a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$$

$$\Rightarrow \left(\frac{1}{a} + \frac{1}{b}\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

$$\Rightarrow \left(\frac{b+a}{ab}\right) \div \left(\frac{b^2-a^2}{a^2b^2}\right)$$

$$\Rightarrow \frac{\frac{(b+a)}{ab}}{\frac{(b^2-a^2)}{a^2b^2}}$$

$$\Rightarrow \frac{(b+a)}{ab} \times \frac{(ab)^2}{(b^2-a^2)}$$

$$\Rightarrow \frac{\cancel{b+a} \cdot (ab)^2}{ab \cdot (\cancel{b+a})(b-a)}$$

$$\Rightarrow \frac{ab}{b-a}$$

Solution - 21

$$(i) (a+b)^{-1} (a^{-1}+b^{-1}) = \frac{1}{ab}$$

$$\text{LHS} \Rightarrow (a+b)^{-1} (a^{-1}+b^{-1})$$

$$\Rightarrow \frac{1}{a+b} \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$\Rightarrow \frac{1}{a+b} \left(\frac{b+a}{ab} \right)$$

$$\Rightarrow \frac{1}{ab} \left(\frac{a+b}{ab} \right)$$

$$\Rightarrow \frac{1}{ab}$$

RHS.

$$(ii) \frac{x+y+z}{x^{-1}y^{-1} + y^{-1}z^{-1} + z^{-1}x^{-1}} = xyz.$$

$$\text{LHS} \Rightarrow \frac{x+y+z}{x^{-1}y^{-1} + y^{-1}z^{-1} + z^{-1}x^{-1}}$$

$$\Rightarrow \frac{x+y+z}{\frac{1}{xy} + \frac{1}{yz} + \frac{1}{zx}}$$

$$\Rightarrow \frac{x+y+z}{\frac{z+xy}{xyz}}$$

$$\Rightarrow \frac{x+y+z}{\frac{(x+y+z)z}{xyz}}$$

$$\Rightarrow xyz$$

\Rightarrow RHS.

Solution - 20

(ii) $\frac{1}{1+a^{m-n}} + \frac{1}{a^{n-m}+1}$

$\rightarrow \frac{1}{1+a^{m-n}} + \frac{1}{1+a^{-(m-n)}}$

$\rightarrow \frac{1}{1+a^{m-n}} + \frac{1}{1+\frac{1}{a^{m-n}}}$

$\rightarrow \frac{1}{1+a^{m-n}} + \frac{1}{\frac{a^{m-n}+1}{a^{m-n}}}$

$\rightarrow \frac{1}{1+a^{m-n}} + \frac{a^{m-n}}{a^{m-n}+1}$

$\rightarrow \frac{1 + a^{m-n}}{1 + a^{m-n}}$

$\rightarrow 1$

Solution - 22

Given $a = c^z$; $b = a^x$; $c = b^y$.

$$\Rightarrow a = c^z$$

$$a = (b^y)^z \quad (\because c = b^y)$$

$$a = b^{yz}$$

$$a = (a^x)^{yz} \quad (\because b = a^x)$$

$$a^1 = a^{xyz}$$

\therefore Bases are equal; so exponents are also equal.

$$\therefore xyz = 1$$

Hence proved.

Solution - 23

Given $a = xy^{p-1}$; $b = xy^{q-1}$; $c = xy^{r-1}$

$$\text{LHS} \Rightarrow a^{q-r} \cdot b^{r-p} \cdot c^{p-q}$$

$$\Rightarrow (xy^{p-1})^{q-r} \cdot (xy^{q-1})^{r-p} \cdot (xy^{r-1})^{p-q}$$

$$\Rightarrow xy^{(p-1)(q-r)} \cdot xy^{(q-1)(r-p)} \cdot xy^{(r-1)(p-q)}$$

$$\Rightarrow xy^{pq-pr-q+r} \cdot xy^{qr-qp-r+p} \cdot xy^{rp-rq-p+q}$$

$$\Rightarrow xy^{p+q+r} \cdot x^{p+q+r} \cdot y^{p+q+r} = x^{p+q+r} \cdot y^{p+q+r} \cdot x^{p+q+r} = x^{p+q+r} \cdot y^{p+q+r} = x^{p+q+r} \cdot y^{p+q+r}$$

$$\Rightarrow xy^0$$

$$\Rightarrow 1$$

\therefore RHS

\therefore Hence proved.

Solution - 24:

$$\text{Given } 2^x = 3^y = 6^{-z}$$

$$\text{let } 2^x = 3^y = 6^{-z} = k$$

$$\Rightarrow 2 = k^{1/x}$$

$$3 = k^{1/y}$$

$$6 = k^{-1/z}$$

$$\frac{1}{6} = k^{1/2}$$

$$\Rightarrow \frac{1}{2 \times 3} = k^{1/2}$$

$$\frac{1}{k^{1/x} \cdot k^{1/y}} = k^{1/2}$$

$$1 = k^{\frac{1}{2}} \cdot k^{\frac{1}{x}} \cdot \frac{1}{y}$$

$$k^0 = k^{\frac{1}{x} + \frac{1}{y} + \frac{1}{2}}$$

$$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{2} = 0 \quad \parallel$$

Solution - 25

Given $2^x = 3^y = 12^z$

let $2^x = 3^y = 12^z = k$

$\Rightarrow 2 = k^{1/x}$

$3 = k^{1/y}$

$12 = k^{1/z}$

$\Rightarrow 2^2 \cdot 3 = k^{1/z}$

$\Rightarrow (k^{1/x})^2 \cdot k^{1/y} = k^{1/z}$

$\Rightarrow k^{\frac{2}{x}} \cdot k^{\frac{1}{y}} = k^{\frac{1}{z}}$

$\Rightarrow \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$

$\Rightarrow \frac{2y + x}{xy} = \frac{1}{z}$

(or) $\frac{2}{x} = \frac{1}{z} - \frac{1}{y}$

$\frac{2}{x} = \frac{y-z}{yz}$

$x = \frac{2yz}{y-z}$

\therefore Hence proved.

Solution - 26 :

$$(i) (3x^2)^0$$

$$\rightarrow 1$$

$$(ii) (xy)^{-2}$$

$$\rightarrow \frac{1}{(xy)^2}$$

$$\Rightarrow \frac{1}{x^2y^2}$$

$$(iii) (-27a^9)^{2/3}$$

$$= (3^3 a^9)^{2/3}$$

$$= (3 \cdot a^3)^{3 \times 2/3}$$

$$= (3a^3)^2$$

$$= 9a^{3 \times 2}$$

$$= 9a^6 //$$

Solution - 27

Given $a=3$; $b=-2$

$$(i) a^a + b^b$$

$$\Rightarrow 3^3 + (-2)^{-2}$$

$$\Rightarrow 3^3 + \frac{1}{(-2)^2}$$

$$\Rightarrow 27 + \frac{1}{4}$$

$$\Rightarrow \frac{108 + 1}{4}$$

$$\Rightarrow \frac{109}{4}$$

$$(ii) a^b + b^a$$

$$\Rightarrow 3^{-2} + (-2)^3$$

$$\Rightarrow \frac{1}{3^2} - 8$$

$$\Rightarrow \frac{1}{9} - 8$$

$$\Rightarrow \frac{1 - 72}{9}$$

$$\Rightarrow \frac{-71}{9}$$

Solution - 28

$$\text{Given } x = 10^3 \times 0.0099 \quad ; \quad y = 10^{-2} \times 110$$

$$\Rightarrow \sqrt{\frac{x}{y}} \Rightarrow \sqrt{\frac{10^3 \times 0.0099}{10^{-2} \times 110}}$$

$$= \sqrt{\frac{10^{3+2} \times 0.0099}{110}}$$

$$\Rightarrow \sqrt{\frac{10^5 \times 0.0099}{110}}$$

$$\Rightarrow \sqrt{\frac{99d}{11d}}$$

$$\rightarrow \sqrt{9}$$

$$\Rightarrow 3$$

Solution - 29:

Given $x=9$, $y=2$, $z=8$

$$x^{1/2} \cdot y^{-1} \cdot z^{2/3}$$

$$\Rightarrow 9^{1/2} \cdot 2^{-1} \cdot 8^{2/3}$$

$$\Rightarrow (3^2)^{1/2} \cdot \left(\frac{1}{2}\right) \cdot (2^3)^{2/3}$$

$$\Rightarrow 3 \cdot \frac{1}{2} \cdot 2^2$$

$$\Rightarrow 3 \cdot \frac{1}{2} \cdot 4^2$$

$$\Rightarrow 6$$

Solution - 30:

Given $x^4 y^2 z^3 = 49392$

$$x^4 y^2 z^3 = 2^4 \cdot 3^2 \cdot 7^4$$

$\therefore x, y, z$ are different primes

$$x=2 ; y=3 ; z=7$$

2	49392
2	24696
2	12348
2	6174
3	3087
3	1029
7	343
7	49
7	7

Solution - 31

$$\text{Given } \sqrt[3]{a^6 b^{-4}} = a^x \cdot b^{2y}$$

$$(a^6 b^{-4})^{1/3} = a^x b^{2y}$$

$$a^{6/3} \cdot b^{-4/3} = a^x b^{2y}$$

$$\begin{aligned} \therefore x &= \frac{6}{3} & ; & \quad 2y = \frac{-4}{3} \\ x &= 2 & & \quad y = \frac{-4}{3 \times 2} \\ & & & \quad y = -2/3 \end{aligned}$$

Solution - 32

$$\text{Given } (P+Q)^{-1} (P^{-1} + Q^{-1}) = P^a \cdot Q^b$$

$$\frac{1}{P+Q} \left(\frac{1}{P} + \frac{1}{Q} \right) = P^a \cdot Q^b$$

$$\frac{1}{P+Q} \left(\frac{Q+P}{QP} \right) = P^a \cdot Q^b$$

$$\frac{1}{QP} = P^a \cdot Q^b$$

$$(QP)^{-1} = P^a \cdot Q^b$$

$$P^{-1} \cdot Q^{-1} = P^a \cdot Q^b$$

$$a = -1$$

$$b = -1$$

$$\therefore \text{LHS} \Rightarrow a + b + 2$$

$$\Rightarrow -1 + 2 - 1$$

$$\Rightarrow 0$$

$$= \text{RHS}$$

Solution- 33

$$\text{Given } \left(\frac{p^{-1} q^2}{p^2 q^{-4}} \right)^7 \div \left(\frac{p^3 q^{-5}}{p^2 q^3} \right)^{-5} = p^x q^y$$

$$\Rightarrow \left(\frac{p^{-7} q^{2 \times 7}}{p^{2 \times 7} q^{-4 \times 7}} \right) \div \left(\frac{p^{3 \times 5} q^{-5 \times 5}}{p^{-2 \times 5} q^{3 \times 5}} \right) = p^x q^y$$

$$\Rightarrow \left(\frac{p^{-7} q^{14}}{p^{14} q^{-28}} \right) \div \left(\frac{p^{15} q^{25}}{p^{10} q^{-15}} \right) = p^x q^y$$

$$\Rightarrow \left(p^{-7-14} q^{14+28} \right) \div \left(p^{15-10} q^{25+15} \right) = p^x q^y$$

$$\Rightarrow \left(p^{-21} q^{42} \right) \div \left(p^5 q^{40} \right) = p^x q^y$$

$$\Rightarrow \left(\frac{p^{-21} q^{42}}{p^5 q^{40}} \right) = p^x q^y$$

$$\Rightarrow (P^{-21-5} \cdot Q^{42-20}) = P^x Q^y$$

$$\Rightarrow (P^{-26} \cdot Q^2) = P^x \cdot Q^y$$

$$\therefore x = -26 ; y = 2$$

$$\begin{aligned} \therefore x + y &= -26 + 2 \\ &= -24. \end{aligned}$$

Solution - 34

$$(i) \quad 5^{2x+3} = 1$$

$$5^{2x+3} = 5^0$$

$$(\because 5^0 = 1)$$

$$\therefore 2x + 3 = 0$$

$$2x = -3$$

$$x = -3/2$$

$$(ii) \quad (13)^{\sqrt{x}} = 4^4 - 3^4 - 6$$

$$(13)^{\sqrt{x}} = 256 - 81 - 6$$

$$(13)^{\sqrt{x}} = 169$$

$$(13)^{\sqrt{x}} = (13)^2$$

$$\Rightarrow (13)^{\sqrt{x}} = 13^2$$

$$\therefore \sqrt{x} = 2$$

$$x = 2^{1/2}$$

$$(iii) \left(\sqrt{\frac{3}{5}} \right)^{x+1} = \frac{125}{27}$$

$$\left(\frac{3}{5} \right)^{\frac{x+1}{2}} = \frac{5^3}{3^3}$$

$$\left(\frac{3}{5} \right)^{\frac{x+1}{2}} = \left(\frac{5}{3} \right)^3$$

$$\left(\frac{3}{5} \right)^{\frac{x+1}{2}} = \left(\frac{3}{5} \right)^{-3}$$

$$\therefore \frac{x+1}{2} = -3$$

$$x+1 = -6$$

$$x = -6-1$$

$$x = -7$$

$$(iv) \quad (3\sqrt{4})^{2x + \frac{1}{2}} = \frac{1}{32}$$

$$\left[(2^2)^{\frac{1}{3}} \right]^{\frac{4x+1}{2}} = \frac{1}{32}$$

$$\left(2^{\frac{2}{3}} \right)^{\frac{4x+1}{2}} = \frac{1}{2^5}$$

$$2^{\frac{4x+1}{3}} = 2^{-5}$$

$$\Rightarrow \frac{4x+1}{3} = -5$$

$$4x + 1 = -15$$

$$4x = -15 - 1$$

$$4x = -16$$

$$x = \frac{-16}{4}$$

$$x = -4 //$$

Solution - 35 :

$$(i) \quad \sqrt{\frac{P}{Q}} = \left(\frac{Q}{P} \right)^{1-2x}$$

$$\left(\frac{P}{Q} \right)^{\frac{1}{2}} = \left(\frac{P}{Q} \right)^{-(1-2x)}$$

$$\left(\frac{P}{Q} \right)^{\frac{1}{2}} = \left(\frac{P}{Q} \right)^{-1+2x}$$

$$\frac{1}{2} = -1 + 2x$$

$$-1 + 2x = \frac{1}{2}$$

$$2x = \frac{1}{2} + 1$$

$$2x = \frac{1+2}{2}$$

$$2x = \frac{3}{2}$$

$$x = \frac{3}{2 \times 2}$$

$$x = \frac{3}{4}$$

$$(ii) \quad 4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^x$$

$$2^{2(x-1)} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^x$$

$$2^{2x-2} \times \frac{1}{2^{3-2x}} = \frac{1}{2^{3x}}$$

$$2^{2x-2} \times 2^{2x-3} = 2^{-3x}$$

$$\therefore 2x-2 + 2x-3 = -3x$$

$$4x-5 = -3x$$

$$4x+3x = 5$$

$$7x = 5$$

$$x = \frac{5}{7} \quad //$$

Solution - 36

Given $5^{3x} = 125$

$$10^y = 0.001$$

$$\Rightarrow 5^{3x} = 125$$

$$5^{3x} = 5^3$$

$$3x = 3$$

$$x = \frac{3}{3}$$

$$x = 1$$

$$\therefore 10^y = 0.001$$

$$10^y = \left(\frac{1}{1000}\right)$$

$$\text{or } 10^y = \left(\frac{1}{10^3}\right)$$

$$10^y = 10^{-3}$$

$$y = -3$$

$$\therefore x = 1 \quad ; \quad y = -3$$

Solution - 37

$$\text{Given } \frac{9^n \cdot 3^2 \cdot 3^n - 27^n}{3^{3m} \cdot 2^3} = \frac{1}{27}$$

$$\frac{3^{2n} \cdot 3^2 \cdot 3^n - 3^{3n}}{3^{3m} \cdot 2^3} = \frac{1}{3^3}$$

$$\frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\frac{3^{3n+2} - 3^{3n}}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot (3^2 - 1)}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot (9 - 1)}{2^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow \frac{3^{3n} \cdot 8}{3^{3m} \cdot 8} = \frac{1}{3^3}$$

$$\Rightarrow 3^{3n} \cdot 3^3 = 3^{3m}$$

∴ 1

$$3^{3(n+1)} = 3^{3m}$$

$$\therefore m = n + 1$$

Solution - 38

$$\text{Given } 3^{4x} = (81)^{-1}$$

$$3^{4x} = (3^4)^{-1}$$

$$3^{4x} = 3^{-4}$$

$$\therefore 4x = -4$$

$$x = \frac{-4}{4}$$

$$x = -1$$

$$\text{and } 10^{\frac{1}{y}} = 0.0001$$

$$10^{\frac{1}{y}} = \frac{1}{10000}$$

$$10^{\frac{1}{y}} = \frac{1}{10^4}$$

$$10^{\frac{1}{y}} = 10^{-4}$$

$$\therefore \frac{1}{y} = -4$$

$$y = -\frac{1}{4}$$

$$\rightarrow 2^{-x} \cdot (16)^y$$

$$\rightarrow 2^{-(-1)} (16)^{-1/4}$$

$$\rightarrow 2^1 \cdot (2^4)^{-1/4}$$

$$\rightarrow 2^1 \cdot 2^{-1}$$

$$\rightarrow 2 \cdot \frac{1}{2}$$

$$\rightarrow 1 //$$

Solution - 39:

$$\text{Given } 3^{x+1} = 9^{x-2}$$

$$3^{x+1} = 3^{2(x-2)}$$

$$3^{x+1} = 3^{2x-4}$$

$$x+1 = 2x-4$$

$$2x-x = 1+4$$

$$x = 5$$

$$2^{1+x} = 2^{1+5}$$

$$= 2^6$$

$$= 64$$

Solution- 40

$$(i) \quad 3(2^x + 1) - 2^{x+2} + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 2^2 + 5 = 0$$

$$3 \cdot 2^x + 3 - 2^x \cdot 4 + 5 = 0$$

$$3 \cdot 2^x - 4 \cdot 2^x + 8 = 0$$

$$-2^x + 8 = 0$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$(ii) \quad 3^x = 9 \cdot 3^y$$

$$3^x = 3^2 \cdot 3^y$$

$$3^x = 3^{2+y}$$

$$x = 2 + y$$

and $8 \cdot 2^y = 4^x$

$$2^3 \cdot 2^y = 2^{2x}$$

$$3 + y = 2x$$

$$x = \frac{3+y}{2} //$$