Chapter 1. Rational and Irrational Numbers

Exercise 1.1

Solution (1)

Given rational numbers are $\frac{2}{9}$ and $\frac{3}{8}$

LCM of denominators 9 and 8 is 72

Equivalent fractions of '$\frac{2}{9}$' and '$\frac{3}{8}$' with denominator 72.

$$\frac{2}{9} = \frac{2\times 8}{9\times 8} = \frac{16}{72} ;$$

$$\frac{3}{8} = \frac{3\times 9}{8\times 9} = \frac{27}{72} .$$

Since, $16 < 27$, $\frac{16}{72} < \frac{27}{72}$

So, $\frac{2}{9} < \frac{3}{8}$

A rational number between $\frac{2}{9}$ and $\frac{3}{8}$ is

$$\frac{\frac{2}{9}+\frac{3}{8}}{2}$$

$$= \frac{\frac{2\times 8+3\times 9}{72}}{2}$$

$$= \frac{16+27}{72\times 2}$$

$$= \frac{43}{144} .$$

Descending order of the numbers is $\frac{3}{8}, \frac{43}{144}, \frac{2}{9}$.

$$\frac{2}{9} < \frac{43}{144} < \frac{3}{8} .$$

Solution (2):

Method II :

L.C.M of 3 and 4 is 12.

Rational number between $\frac{1}{3}$ and $\frac{1}{4}$ is $\dfrac{\frac{1}{3}+\frac{1}{4}}{2}$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12 \times 2}$$

$$= \frac{7}{24}$$

$\frac{1}{3} = \frac{1\times4}{3\times4} = \frac{4}{12}$;

$\frac{1}{4} = \frac{1\times3}{4\times3} = \frac{3}{12}$.

Since, $4 > 3$. $\frac{4}{12} > \frac{3}{12}$

$\frac{1}{3} > \frac{1}{4}$

'$\frac{7}{24}$' is in between $\frac{1}{3}$ and $\frac{1}{4}$. So, $\frac{1}{3} > \frac{7}{24} > \frac{1}{4} \rightarrow$ (1)

Rational number between $\frac{1}{3}$ and $\frac{7}{24}$ is $\dfrac{\frac{1}{3}+\frac{7}{24}}{2}$

$$= \frac{\frac{1\times8+7\times1}{24}}{2}$$

$$= \frac{15}{2\times24}$$

$$= \frac{15}{48}$$

$\frac{15}{48}$ is in between $\frac{1}{3}$ and $\frac{7}{24}$. So, $\frac{1}{3} > \frac{15}{48} > \frac{7}{24} \rightarrow$ (2)

From (1) and (2) , ascending order of numbers (increasing order)

is $\frac{1}{4} < \frac{7}{24} < \frac{15}{48} < \frac{1}{3}$

Solution 3

Given rational numbers are $-\frac{1}{3}$ and $-\frac{1}{2}$

LCM of 3 and 2 is 6.

$-\frac{1}{3} = \frac{-1\times2}{3\times2} = \frac{-2}{6}$; $-\frac{1}{2} = \frac{-1\times3}{2\times3} = \frac{-3}{6}$

Since, $2 < 3$

$-2 > -3$

$\frac{-2}{6} > \frac{-3}{6}$

So, $\frac{-1}{3} > \frac{-1}{2}$

Rational number between $\frac{-1}{3}$ and $\frac{-1}{2}$ is $\dfrac{\frac{-1}{3} + \left(\frac{-1}{2}\right)}{2}$

$= \dfrac{\frac{-1\times2 + (-1)(3)}{6}}{2}$

$= \frac{-2-3}{6\times2}$

$= \frac{-5}{12}$

$\therefore \frac{-1}{3} > \frac{-5}{12} > \frac{-1}{2} \rightarrow 1$

Rational number between $\frac{-1}{3}$ and $\frac{-5}{12}$ is $\dfrac{\frac{-1}{3} + \left(\frac{-5}{12}\right)}{2}$

$= \dfrac{\frac{-1\times4 + (-5)(1)}{12}}{2}$

$= \frac{-4-5}{12\times2}$

$= \frac{-9}{24}$

$\therefore \frac{-1}{3} > \frac{-9}{24} > \frac{-5}{12} \rightarrow 2$

From 1 and 2, $\frac{-1}{3} > \frac{-9}{24} > \frac{-5}{12} > \frac{-1}{2}$

Ascending order (increasing order) of rational numbers

$\frac{-1}{2}, \frac{-5}{12}, \frac{-9}{24}, \frac{-1}{3}$.

Solution (4):

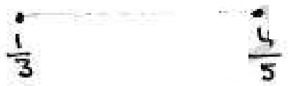
LCM of 3 and 5 is 15.

$\frac{1}{3} = \frac{1\times5}{3\times5} = \frac{5}{15}$; $\frac{4}{5} = \frac{4\times3}{5\times3} = \frac{12}{15}$

Since, $5 < 12$

So, $\frac{5}{15} < \frac{12}{15}$

$\frac{1}{3} < \frac{4}{5}$



Rational number between $\frac{1}{3}$ and $\frac{4}{5}$ is $\dfrac{\frac{1}{3}+\frac{4}{5}}{2}$

$= \dfrac{\frac{1\times5+4\times3}{3\times5}}{2}$

$= \dfrac{\frac{5+12}{15}}{2}$

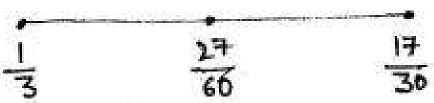
$= \frac{17}{30}$

Rational number between '$\frac{1}{3}$' and '$\frac{17}{30}$' is

$\dfrac{\frac{1}{3}+\frac{17}{30}}{2}$

$= \dfrac{\frac{1\times10+17}{30}}{2}$

$= \frac{27}{60}$



Rational number between '$\frac{17}{30}$' and '$\frac{4}{5}$' is

$$\frac{\frac{17}{30} + \frac{4}{5}}{2}$$

$$= \frac{\frac{17 + 4\times 6}{30}}{2}$$

$$= \frac{41}{60}$$

$$\frac{17}{30} \qquad \frac{41}{60} \qquad \frac{4}{5}$$

→ Rational numbers between $\frac{1}{3}$ and $\frac{4}{5}$ are

$$\frac{1}{3} \quad \frac{27}{60} < \frac{17}{30} < \frac{41}{60} < \frac{4}{5}$$

Descending order (decreasing order) of numbers are

$$\frac{4}{5}, \frac{41}{60}, \frac{17}{30}, \frac{27}{60}, \frac{1}{3}.$$

Solution (5):

A rational number between 4 and 4.5 is $\frac{4+4.5}{2} = \frac{8.5}{2}$

$= 4.25$

A rational number between 4 and 4.25 = $\frac{4+4.25}{2} = \frac{8.25}{2}$

$= 4.125$

A rational number between 4 and 4.125 = $\frac{4+4.125}{2} = \frac{8.125}{2}$

$= 4.0625$

three rational numbers between 4 and 4.5 are

4.0625, 4.125, 4.25 ,,

Solution (6) :

We need to insert Six rational numbers between 3 and 4. So, we multiply both numerator and denominators of rational numbers with 6+1 i.e., 7.

So, $\frac{3}{1} = \frac{3\times7}{1\times7} = \frac{21}{7}$

$\frac{4}{1} = \frac{4\times7}{1\times7} = \frac{28}{7}$

We have, $21 < 22 < 23 < 24 < 25 < 26 < 27$

$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$

therefore, six rational numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.

Solution (7) :

We need to insert five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. So, we multiply both numerator and denominator with 5+1 i.e., 6.

So, $\frac{3}{5} = \frac{3\times6}{5\times6} = \frac{18}{30}$

$\frac{4}{5} = \frac{4\times6}{5\times6} = \frac{24}{30}$

We have, $18 < 19 < 20 < 21 < 22 < 23 < 24$

$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$

therefore, five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

$\frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30}$.

Solution (8) :

LCM of 5 and 7 is 35.

$\frac{-2}{5} = \frac{-2\times7}{5\times7} = \frac{-14}{35}$; $\frac{1}{7} = \frac{1\times5}{7\times5} = \frac{5}{35}$.

We need to insert ten rational numbers between $\frac{-2}{5}\left(=\frac{-14}{35}\right)$ and $\frac{1}{7}\left(=\frac{5}{35}\right)$. So, we can select any ten numbers between -14 and 5 as numerators and '35' as denominator.

$\therefore$ $\frac{-13}{35}, \frac{-12}{35}, \frac{-11}{35}, \frac{-10}{35}, \frac{-9}{35}, \frac{-8}{35}, \frac{-7}{35}, \frac{-6}{35}, \frac{-5}{35}, \frac{-4}{35}$ are ten rational numbers which are in between $\frac{-2}{5}$ and $\frac{1}{7}$.

Solution (9)

LCM of 2 and 3 is 6.

$\frac{1}{2} = \frac{1\times3}{2\times3} = \frac{3}{6}$; $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$

We need to insert six rational numbers. So, multiply both numerator and denominator by 6+1 i.e, 7.

$\frac{3}{6} = \frac{3\times7}{6\times7} = \frac{21}{42}$;

$\frac{4}{6} = \frac{4\times7}{6\times7} = \frac{28}{42}$

Since, $21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$

$\frac{21}{42} < \frac{22}{42} < \frac{23}{42} < \frac{24}{42} < \frac{25}{42} < \frac{26}{42} < \frac{27}{42} < \frac{28}{42}$

Therefore, six numbers between $\frac{1}{2}$ and $\frac{2}{3}$ are $\frac{22}{42}, \frac{23}{42}, \frac{24}{42}, \frac{25}{42}, \frac{26}{42}, \frac{27}{42}$,

Exercise - 1.2

Solution 1:

Let $\sqrt{5}$ be a rational number, then

$\sqrt{5} = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factors (except 1).

$$\Rightarrow 5 = \frac{p^2}{q^2}$$

$$p^2 = 5.q^2 \longrightarrow (i)$$

As '5' divides $5q^2$, so '5' divides p^2 and '5' is prime

$\Rightarrow$ 5 divides p.

Let $p = 5.m$, where m is an integer

Substituting the value of 'p' in (i)

$$(5m)^2 = 5.q^2$$

$$25.m^2 = 5.q^2$$

$$\Rightarrow q^2 = 5.m^2$$

As 5 divides $5m^2$, so 5 divides q^2 but 5 is prime

$\Rightarrow$ 5 divides q.

thus, p and q have a common factor 5. This contradicts that 'p' and 'q' have no common factors (except 1).

Hence, $\sqrt{5}$ is not rational number.

So, we conclude $\sqrt{5}$ is a irrational number.

Solution 2:

Let $\sqrt{7}$ be a rational number.

$\sqrt{7} = \frac{p}{q}$, where p, q are integers, $q \neq 0$ and p, q have no common factors (Except 1).

$$\Rightarrow 7 = \frac{p^2}{q^2}$$

$$p^2 = 7 \cdot q^2 \longrightarrow (i)$$

As 7 divides $7q^2$, so 7 divides p^2 and 7 is a prime

$\Rightarrow$ 7 divides p

Let ~~7 p^2 = 7.q^2~~ $p = 7.m$, where 'm' is an integer

Substitute this value of 'p' in (i) we have.

$$(7m)^2 = 7 \cdot q^2$$

$$49 \cdot m^2 = 7 \cdot q^2$$

$$q^2 = 7 \cdot m^2$$

As 7 divides $7m^2$, so 7 divides q^2 but 7 is a prime number.

$\Rightarrow$ 7 divides q

Thus, 'p' and q have a common factor 7. This contradicts that p and q have no common fact (except 1).

Hence, $\sqrt{7}$ is not a rational number.

So, we conclude $\sqrt{7}$ is irrational number.

Solution 3:

Let $\sqrt{6}$ be a rational number.

$\sqrt{6} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (Except 1).

$\Rightarrow 6 = \frac{p^2}{q^2}$

$p^2 = 6.q^2 \longrightarrow$ (i)

As '2' divides $6q^2$, so 2 divides p^2 but 2 is prime

$\Rightarrow$ 2 divides p.

Let $p = 2.m$, where 'm' is an integer.

Substitute this value of 'p' in (i)

$(2m)^2 = 6.q^2$

$4.m^2 = 6.q^2$

$2.m^2 = 3.q^2$

'2' divides '$2m^2$', so 2 divides '$3q^2$'

2 should either divide '3' or divide q^2.

But 2 ~~should~~ does not divide 3.

Therefore, 2 divides q^2 and 2 is a prime

2 divides q.

Thus, p and q have a common factor 2. This contradicts that 'p' and 'q' have no common factors (Except 1).

Hence, $\sqrt{6}$ is not rational number.

So, we conclude $\sqrt{6}$ is irrational number.

Solution 4 :-

Let $\frac{1}{\sqrt{11}}$ be a rational number.

$\frac{1}{\sqrt{11}} = \frac{p}{q}$, where 'p', 'q' are integers, $q \neq 0$ and p, q have no common factors (except 1).

$\Rightarrow \frac{1}{11} = \frac{p^2}{q^2}$

$\Rightarrow q^2 = 11 \cdot p^2 \longrightarrow$ (i)

As 11 divides $11p^2$, so 11 divides q^2 but 11 is a prime.

$\Rightarrow$ 11 divides q.

$q = 11m$, where 'm' is an integer.

$(11m)^2 = 11p^2$

$\Rightarrow 121 \cdot m^2 = 11 \cdot p^2$

$\Rightarrow p^2 = 11 \cdot m^2$

As 11 divides $11m^2$, so 11 divides p^2 but 11 is a prime.

$\Rightarrow$ 11 divides p.

Thus, p and q have a common factor 11. This contradicts the fact that p and q has no common factors (Except 1).

Hence, $\frac{1}{\sqrt{11}}$ is not rational number.

So, we conclude $\frac{1}{\sqrt{11}}$ is irrational number.

Solution 5 :

Let '$\sqrt{2}$' is a rational number.

$\sqrt{2} = \frac{p}{q}$. where p and q are integers, $q \neq 0$ and p, q have no common factors (Except 1).

$\Rightarrow 2 = \frac{p^2}{q^2}$

$p^2 = 2 \cdot q^2 \longrightarrow (i)$

As '2' divides $2q^2$, so 2 divides p^2 but 2 is prime.

$\Rightarrow$ 2 divides p.

Let $p = 2m$, where 'm' is an integer.

Substitute this value of p in (i).

$(2m)^2 = 2q^2$

$4m^2 = 2q^2$

$\Rightarrow q^2 = 2m^2$

As 2 divides $2m^2$, so 2 divides q^2 but 2 is prime.

2 divides q

Thus, p and q have a common factor 2. This contradicts the fact that p and q has no common factor (Except 1).

Hence, $\sqrt{2}$ is not rational number.

So, we conclude $\sqrt{2}$ is irrational number.

Let us assume $3-\sqrt{2}$ is rational number, say r.

Thus, $3-\sqrt{2} = r \Rightarrow 3 - r = \sqrt{2}$

As 'r' is rational, $3-r$ is rational $\Rightarrow \sqrt{2}$ is rational

this contradicts the fact that $\sqrt{2}$ is irrational

Hence, our assumption is wrong. Therefore, $3-\sqrt{2}$ is an irrational number.

Solution 6 :

Let $\sqrt{3}$ is a rational number.

$\sqrt{3} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (Except 1).

$$3 = \frac{p^2}{q^2}$$

$$\Rightarrow p^2 = 3q^2 \longrightarrow (i)$$

As '3' divides $3q^2$, so 3 divides p^2 but 3 is a prime.

$\Rightarrow$ 3 divides p

Let $p = 3.m$, where 'm' is an integer

substituting this value of p in (i).

$$(3m)^2 = 3q^2$$

$$9m^2 = 3q^2$$

$$\Rightarrow q^2 = 3m^2$$

As '3' divides $3m^2$, so '3' divides q^2 but 3 is not prime.

$\Rightarrow$ 3 divides q

Thus, p and q have a common factor 3. This contradicts the fact that p and q has no common factor (Except 1).

Hence, $\sqrt{3}$ is not rational number.

So, we conclude $\sqrt{3}$ is irrational number.

Let us assume $\frac{2}{5}\sqrt{3}$ is a rational number, say r.

Thus $\frac{2}{5}\sqrt{3} = r \Rightarrow \sqrt{3} = \frac{5}{2}.r$

As r is rational, $\frac{5}{2}r$ is rational $\Rightarrow \sqrt{3}$ is rational

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $\frac{2}{5}\sqrt{3}$ is irrational number.

Solution 7 :

Let $\sqrt{5}$ is a rational number.

$\sqrt{5} = \frac{p}{q}$, where p and q are integers, $q \neq 0$ and p, q have no common factors (Except 1).

$5 = \frac{p^2}{q^2}$

$\Rightarrow p^2 = 5.q^2 \longrightarrow (i)$

As 5 divides $5q^2$, so 5 divides p^2 but 5 is a prime.

$\Rightarrow$ 5 divides p.

Let $p = 5m$ where m is an integer

Substitute this value of m in (i)

$(5m)^2 = 5q^2$

$25m^2 = 5.q^2$

$q^2 = 5m^2$

As 5 divides $5m^2$, 5 divides q^2 but 5 is a prime.

5 divides q.

Thus, p and q have a common factor 5. This contradicts the fact that p and q has no common factors (Except 1).

Hence, $\sqrt{5}$ is not rational number.

So, we conclude $\sqrt{5}$ is irrational number.

Let us assume $-3+2\sqrt{5}$ is a rational number, say r.

Thus, $-3+2\sqrt{5} = r \Rightarrow -3-r = 2\sqrt{5}$

$\Rightarrow \sqrt{5} = \frac{-(3+r)}{2}$

As 'r' is rational, $-\left(\frac{3+r}{2}\right)$ is rational $\Rightarrow \sqrt{5}$ is rational

This contradict the fact that $\sqrt{5}$ is irrational.

Hence, our assumption is wrong, therefore, $-3+2\sqrt{5}$ is irrational number.

Solution (8) :

(i) Let '$5+\sqrt{2}$' is rational number, Say r.

$$5+\sqrt{2} = r \Rightarrow \sqrt{2} = r-5$$

As 'r' is rational, $r-5$ is rational $\Rightarrow \sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is wrong. Therefore, $5+\sqrt{2}$ is an irrational number.

(ii)

Let $3-5\sqrt{3}$ is rational number, say r.

$$3-5\sqrt{3} = r \Rightarrow 5\sqrt{3} = 3-r$$

$$\Rightarrow \sqrt{3} = \left(\frac{3-r}{5}\right)$$

As r is rational, $\left(\frac{3-r}{5}\right)$ is rational $\Rightarrow \sqrt{3}$ is rational

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $3-5\sqrt{3}$ is an irrational number.

(iii) Let $2\sqrt{3}-7$ is a rational number, say r.

$$2\sqrt{3}-7 = r \Rightarrow 2\sqrt{3} = r+7$$

$$\sqrt{3} = \frac{r+7}{2}$$

As 'r' is rational, $\left(\frac{r+7}{2}\right)$ is rational $\Rightarrow \sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

Hence, our assumption is wrong. Therefore, $2\sqrt{3}-7$ is an irrational number.

<u>Solution 8</u> :

(iv) Let $\sqrt{2}+\sqrt{5}$ is a rational number, say r.

$$\sqrt{2}+\sqrt{5} = r$$

$$\sqrt{5} = r - \sqrt{2}$$

$$(\sqrt{5})^2 = (r-\sqrt{2})^2 \quad \text{(on Squaring both Sides)}$$

$$5 = r^2 + (\sqrt{2})^2 - 2 \times r \times \sqrt{2} \qquad \left[\because (a-b)^2 = a^2+b^2-2ab\right]$$

$$5 = r^2 + 2 - 2\sqrt{2}\cdot r$$

$$2\sqrt{2}\cdot r = r^2 - 3$$

$$\sqrt{2} = \frac{r^2-3}{2\cdot r}$$

As r is rational, r^2-3 is rational, $\left(\frac{r^2-3}{2r}\right)$ is rational

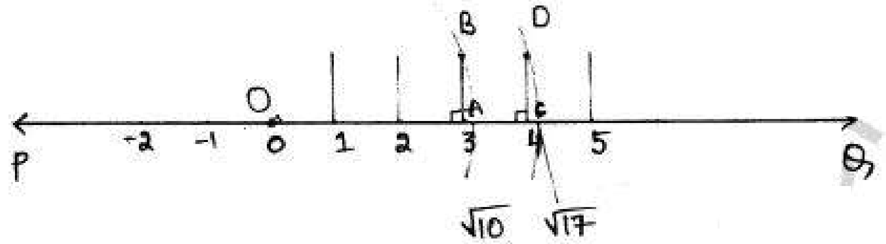
$\Longrightarrow \sqrt{2}$ is rational

this contradicts the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is wrong. therefore, $(\sqrt{2}+\sqrt{5})$ is an irrational number.

Exercise 1.3

solution 1 :



PQ is a number line.

We have two right angle triangles. They are

ΔOAB and ΔOCD.

In a right angled triangle,

$(\text{hypotenuse})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$

$\therefore \quad OB^2 = OA^2 + AB^2$

$OB^2 = 3^2 + 1^2 \qquad \left(\because \begin{matrix} OA = 3 \\ AB = 1 \end{matrix}\right)$

$OB^2 = 9 + 1$

$OB = \sqrt{10}$

Similarly, in ΔOCD,

$OD^2 = OC^2 + CD^2$

$OD^2 = 4^2 + 1^2 \qquad \left(\because \begin{matrix} OC = 4 \\ CD = 1 \end{matrix}\right)$

$OD^2 = 16 + 1$

$OD = \sqrt{17}$

Solution 2 :-

(i) $\frac{36}{100}$

$$
\begin{array}{r|l}
 & 0.036 \\
\hline
100 & 360 \\
 & 300 \\
\hline
 & \;\;600 \\
 & \;\;600 \\
\hline
 & \;\;\;\;0
\end{array}
$$

Remainder becomes zero.

Decimal Expansion of $\frac{36}{100}$ $(= 0.36)$ is terminating.

(ii) $4\frac{1}{8}$

$= \frac{4 \times 8 + 1}{8}$

$= \frac{33}{8}$

$$
\begin{array}{r|l}
 & 4.125 \\
\hline
8 & 33 \\
 & 32 \\
\hline
 & \;\;10 \\
 & \;\;\;8 \\
\hline
 & \;\;20 \\
 & \;\;16 \\
\hline
 & \;\;\;\;40 \\
 & \;\;\;\;40 \\
\hline
 & \;\;\;\;\;0
\end{array}
$$

Remainder becomes zero

Decimal expansion of $4\frac{1}{8}$ $(= 4.125)$ is terminating.

(iii) $\frac{2}{9}$

$$
\begin{array}{r|l}
 & 0.22 \\
\hline
9 & 20 \\
 & 18 \\
\hline
 & \;20 \\
 & \;18 \\
\hline
 & \;\;2 \leftarrow \text{Remainder is repeating.}
\end{array}
$$

In the above decimal Expansion remainder is repeating. So, it is a non-terminating decimal.

So, $\frac{2}{9} = 0.222\ldots = 0.\dot{2} = 0.\bar{2}$

(iv) $\frac{2}{11}$

$$
\begin{array}{r|l}
 & 0.1818 \\
\hline
11 & 20 \\
 & 11 \\
\hline
 & \;90 \\
 & \;88 \\
\hline
 & \;\;20 \\
 & \;\;11 \\
\hline
 & \;\;\;90 \\
 & \;\;\;88 \\
\hline
 & \;\;\;\;2 \leftarrow \text{Remainder '2' is repeating.}
\end{array}
$$

Decimal Expansion of $\frac{2}{11} = 0.181818\ldots\ldots$

Here, remainder is repeating. So, it a non-terminating repeating decimal.

$\therefore \frac{2}{9} = 0.\overline{18}$

(v) $\frac{3}{13}$

$$
\begin{array}{r|l}
 & 0.230769 \\
\hline
13 & 30 \\
 & 26 \\
\hline
 & \quad 40 \\
 & \quad 39 \\
\hline
 & \quad\ 100 \\
 & \quad\ \ 91 \\
\hline
 & \qquad 90 \\
 & \qquad 78 \\
\hline
 & \qquad 120 \\
 & \qquad 117 \\
\hline
 & \qquad\quad 3
\end{array}
\longleftarrow \text{Remainder '3' is repeating.}
$$

Decimal Expansion of $\frac{3}{13}$ is $0.230769......$

Here, remainder is repeating. So, it a non-terminating repeating decimal.

$\therefore \ \frac{3}{13} = 0.\overline{230769}$

(vi) $\frac{329}{400}$

$$
\begin{array}{r|l}
 & 0.8225 \\
\hline
400 & 3290 \\
 & 3200 \\
\hline
 & \quad 900 \\
 & \quad 800 \\
\hline
 & \quad 1000 \\
 & \quad\ 800 \\
\hline
 & \quad\ 2000 \\
 & \quad\ 2000 \\
\hline
 & \qquad\ 0
\end{array}
\longleftarrow \text{Remainder is zero}
$$

Decimal Expansion of $\frac{329}{400} = 0.8225$.

$\therefore$ It is a ~~repeat~~ terminating decimal Expansion.

Solution ③ :-

(i) $\frac{13}{3125}$

Prime factorization of denominator 3125.

$$3125 = 5 \times 5 \times 5 \times 5 \times 5 \times 1$$
$$= 5^5 \times 1$$
$$= 1 \times 5^5$$

$$\begin{array}{r|c} 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore \; 3125 = 2^0 \times 5^5 \quad (\because 2^0 = 1)$

Since, denominator is in the form of $2^0 \times 5^5$, the decimal Expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

Prime factorization of denominator 8.

$$8 = 2 \times 2 \times 2$$
$$8 = 2^3 \times 1$$
$$8 = 2^3 \times 5^0$$

$$\begin{array}{r|c} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Since denominator is in the form of $2^3 \times 5^0$, decimal Expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{23}{75}$

Prime factorization of 75.

$$75 = 3 \times 5 \times 5 \times 1$$

$$75 = 3 \times 5^2 \times 1$$

$$75 = 3 \times 2^0 \times 5^2 \quad (\because 2^0 = 1)$$

$$\begin{array}{r|l} 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Since, denominator contains prime factor 3 other than 2 or 5.

Decimal Expansion of $\frac{23}{75}$ is non-terminating.

(iv) $\frac{6}{15}$

Both numerator and denominator contains common factor 3.

$$\frac{6}{15} = \frac{3 \times 2}{3 \times 5} = \frac{2}{5}$$

$$\therefore \frac{6}{15} = \frac{2}{5}$$

Since, denominator is in the form $2^0 \times 5^1$.

Decimal Expansion of $\frac{6}{15}\left(=\frac{2}{5}\right)$ is terminating.

(v) $\frac{1258}{625}$

Prime factorization of denominator 625.

$$625 = 5 \times 5 \times 5 \times 5 \times 1$$

$$625 = 5^4 \times 2^0$$

$$\begin{array}{r|l} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1. \end{array}$$

Since, denominator is in the form $2^0 \times 5^4$, decimal Expansion of $\frac{1258}{625}$ is terminating.

(vi) $\frac{77}{210}$

Both numerator and denominator contains common factor 7.

$$\frac{77}{210} = \frac{7 \times 11}{7 \times 30} = \frac{11}{30}$$

$$\therefore \frac{77}{210} = \frac{11}{30}$$

Prime factorization of denominator 30.

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$30 = 2 \times 3 \times 5 \times 1$$

$$30 = 3 \times 2 \times 5$$

Since, denominator contains prime factor 3 other 2 or 5

Decimal expansion of $\frac{77}{210}$ is non-terminating.

Solution (4) :-

Expressing both numerator and denominator of fraction $\frac{987}{10500}$ as product of prime numbers by prime factorization method

$$\begin{array}{r|l} 3 & 987 \\ \hline 7 & 329 \\ \hline 47 & 47 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 10500 \\ \hline 2 & 5250 \\ \hline 3 & 2625 \\ \hline 5 & 875 \\ \hline 5 & 175 \\ \hline 5 & 35 \\ \hline & 7 \end{array}$$

$$\therefore 987 = 3 \times 7 \times 47$$

$$\therefore 10500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5 \times 7$$

$$\frac{987}{10500} = \frac{\cancel{3}\times\cancel{7}\times 47}{2\times2\times\cancel{3}\times5\times5\times5\times\cancel{7}}$$

$$= \frac{47}{2^2\times5^3}$$

Since, denominator is in the form $2^2\times5^3$, decimal Expansion of $\frac{987}{10500}$ is terminating

Solution 5 :—

(i) $\frac{17}{8}$

Prime factorization of denominator 8

$$8 = 2\times2\times2\times1$$

$$8 = 2^3\times5^0 \qquad (\because a^0 = 1)$$

$$\begin{array}{r|l} 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

$$\frac{17}{8} = \frac{17}{2^3}$$

$$= \frac{17\times5^3}{2^3\times5^3}$$ (By multiplying both numerator and denominator with 5^3).

$$= \frac{17\times125}{(2\times5)^3}$$

$$\begin{array}{r} 125 \\ \times 17 \\ \hline 875 \\ 125\times \\ \hline 2125 \end{array}$$

$$= \frac{2125}{10^3}$$

$= 2.125$ (Since, denominator is in the form 10^3, decimal Expansion is obtained by moving decimal point to three digits from right).

$\therefore \frac{17}{8} = 2.125$

(ii) $\frac{13}{3125}$

Prime factorization of 3125.

$$3125 = 5\times5\times5\times5\times5 = 5^5$$

$$\begin{array}{r|l} 5 & 3125 \\ \hline 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\frac{13}{3125} = \frac{13}{5^5}$$

$$= \frac{13\times2^5}{5^5\times2^5} \quad \left(\text{Multiplying numerator and denominator by } 2^5\right)$$

$$= \frac{13\times32}{(2\times5)^5} \quad \left(\begin{array}{c} 2^5 = 2\times2\times2\times2\times2 = 32 \\ \& \ a^m\times b^m = (a\times b)^m \end{array}\right)$$

$$= \frac{416}{10^5}$$

$$\begin{array}{r} 32 \\ \times13 \\ \hline 96 \\ 32\times \\ \hline 416 \end{array}$$

$$= 0.00416$$

$$\therefore \frac{13}{3125} = 0.00416$$

(iii) $\frac{7}{80}$

Prime factorization of 80.

$$80 = 2\times2\times2\times2\times5$$

$$80 = 2^4\times5^1$$

$$\begin{array}{r|l} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$\frac{7}{80} = \frac{7}{2^4\times5^1}$$

$$= \frac{7\times5^3}{2^4\times5^1\times5^3} \quad \left(\text{Multiplying numerator and denominator by } 5^3\right)$$

$$= \frac{7\times125}{2^4\times5^4}$$

$= \dfrac{7 \times 125}{(2 \times 5)^4}$

$$\begin{array}{r} \overset{1}{1}\overset{3}{2}5 \\ \times 7 \\ \hline 875 \end{array}$$

$= \dfrac{875}{10^4}$

$= 0.0875$ (Since, denominator is 10^4, decimal Expansion can be obtained by moving decimal point of numerator to four digits from right.)

$\therefore \dfrac{7}{80} = 0.0875$

(iv) $\dfrac{6}{15}$

Prime factorization of 6 and 15

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore 6 = 2 \times 3$

$\therefore 15 = 3 \times 5$

$\dfrac{6}{15} = \dfrac{2 \times \cancel{3}}{\cancel{3} \times 5} = \dfrac{2}{5}$

$= \dfrac{2 \times 2}{5 \times 2}$ (By multiplying both numerator and denominator by 2)

$= \dfrac{4}{10}$

$= 0.4$

(v) $\frac{2^2 \times 7}{5^4}$

$= \frac{2^2 \times 7 \times 2^4}{5^4 \times 2^4}$ (By multiplying both numerator and denominator by 2^4)

$= \frac{4 \times 7 \times 16}{(2 \times 5)^4}$

$$\begin{array}{r} 28 \\ \times 16 \\ \hline 168 \\ 28\times \\ \hline 448 \\ \hline \end{array}$$

$= \frac{28 \times 16}{10^4}$

$= \frac{448}{10^4}$

$= 0.0448$

$\therefore \frac{2^2 \times 7}{5^4} = 0.0448$

(vi) $\frac{237}{1500}$

Prime factorization of 237 and 1500.

$$\begin{array}{r|l} 3 & 237 \\ \hline 79 & 79 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 1500 \\ \hline 2 & 750 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore 237 = 3 \times 79$ $\qquad$ $\therefore 1500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5$

$\frac{237}{1500} = \frac{\cancel{3} \times 79}{2^2 \times \cancel{3} \times 5^3}$

$= \frac{79}{2^2 \times 5^3}$

$$= \frac{79 \times 2}{2^2 \times 5^3 \times 2}$$ (Multiplying both numerator and denominator by '2'.)

$$= \frac{158}{2^3 \times 5^3}$$

$$= \frac{158}{(10)^3}$$

$$= 0.158$$

$$\therefore \frac{237}{1500} = \underline{\underline{0.158}}$$

Solution ⑥:

Given rational number $\frac{257}{5000}$

Prime factorization of denominator 5000

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$$\therefore 5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$$
$$= 2^3 \times 5^4$$

Thus, denominator of rational number is in the form $2^m \times 5^n$, where $m=3$ and $n=4$.

$$\therefore \frac{257}{5000} = \frac{257}{2^3 \times 5^4}$$

$$= \frac{257 \times 2}{2^3 \times 5^4 \times 2}$$ (By multiplying numerator and denominator by 2)

$$= \frac{514}{2^4 \times 5^4}$$

$$= \frac{514}{(2\times5)^4}$$

$$= \frac{514}{10^4}$$

$$= 0.0514$$

$\therefore$ Decimal expansion of $\frac{257}{5000}$ is $\underline{\underline{0.0514}}$.

Solution (7) :-

Decimal expansion of $\frac{1}{7}$:

$$
\begin{array}{r|l}
 & 0.142857 \\
\hline
7 & 10 \\
 & \underline{7} \\
 & 30 \\
 & \underline{28} \\
 & 20 \\
 & \underline{14} \\
 & 60 \\
 & \underline{56} \\
 & 40 \\
 & \underline{35} \\
 & 50 \\
 & \underline{49} \\
 & 1
\end{array}
\quad \longleftarrow \text{Remainder '1' is repeated.}
$$

$\therefore$ Decimal Expansion of $\frac{1}{7}$ is non-terminating repeating.

$$\frac{1}{7} = 0.\overline{142857}$$

$\frac{2}{7}$ Can be written as $2\times\frac{1}{7}$

Decimal Expansion of $\frac{2}{7} = 2\times\frac{1}{7}$

$= 2\times 0.\overline{142857}$

$= 0.\overline{285714}$

Similarly,

$\frac{3}{7} = 3\times\frac{1}{7} = 3\times 0.\overline{142857}$

$= 0.\overline{428571}$

$\frac{4}{7} = 4\times\frac{1}{7} = 4\times 0.\overline{142857}$

$= 0.\overline{571428}$

$\frac{5}{7} = 5\times\frac{1}{7} = 5\times 0.\overline{142857}$

$= 0.\overline{714285}$

$\frac{6}{7} = 6\times\frac{1}{7} = 6\times 0.\overline{142857}$

$= 0.\overline{857142}$

Solution (8) :-

(i) Let $x = 0.\overline{3} = 0.3333........ \rightarrow (1)$

As there is one repeating digit after the decimal point so multiplying both sides of Eq (1) by 10.

$10x = 3.333..... \rightarrow (2)$

Subtracting (1) from (2), we get.

$10x - x = 3.333.... - 0.333....$

$$9x = 3$$

$$x = \frac{3}{9}$$

$$x = \frac{1}{3}$$

$\therefore \; x = 0.\overline{3} = \frac{1}{3}$, which is in $\frac{p}{q}$ form.

(ii) Let $x = 5.\overline{2} = 5.222\ldots \rightarrow ①$

As there is one repeating digit after the decimal point, so multiplying both sides of Eq ① by 10.

$$10x = 52.222\ldots \rightarrow ②$$

Subtracting ① from ②, we get

$$\begin{aligned} 10x &= 52.222\ldots \\ (-) \quad x &= 5.222\ldots \\ \hline 9x &= 47.000 \end{aligned}$$

$$x = \frac{47}{9}$$

$\therefore \; x = 5.\overline{2} = \frac{47}{9}$, which is in $\frac{p}{q}$ form.

(iii) Let $x = 0.404040\ldots \rightarrow ①$

As there is two repeating digit after the decimal point, so multiolying both sides of Eq ① by 100

$$100x = 40.404040 \rightarrow ②$$

Subtracting ① from ②, we get.

$$\begin{aligned} 100x &= 40.404040\ldots \\ (-) \quad x &= 0.404040\ldots \\ \hline 99x &= 40.000 \end{aligned}$$

$$99x = 40$$

$$x = \frac{40}{99}$$

$\therefore x = 0.404040\ldots = \frac{40}{99}$, which is in $\frac{p}{q}$ form.

(iv) Let $x = 0.4\overline{7} = 0.4777\ldots$

$$x = 0.4777\ldots \longrightarrow (1)$$

There is one non-repeating digit after the decimal point, multiplying both sides of (1) by 10.

$$10x = 4.777\ldots \longrightarrow (2)$$

As there is one repeating digit after the decimal point, multiplying both sides of (2) by 10.

$$100x = 47.777\ldots \longrightarrow (3)$$

Subtracting (2) from (3), we get.

$$\begin{array}{rl} 100x = & 47.777\ldots \\ 10x = & 4.777\ldots \\ (-) & \\ \hline 90x = & 43.000 \end{array}$$

$$90x = 43$$

$$x = \frac{43}{90}$$

$\therefore x = 0.4777\ldots = \frac{43}{90}$, which is in $\frac{p}{q}$ form.

(v) $0.1\overline{34}$

Let $x = 0.134343434 \ldots\ldots \rightarrow (1)$

There is one non-repeating digit after the decimal point, multiplying both sides of (1) by 10.

$$10x = 1.343434 \ldots \rightarrow (2)$$

As there are two repeating digits after the decimal point, so multiplying both sides of (2) by 100.

$$1000x = 134.343434 \ldots \rightarrow (3)$$

Subtracting (2) from (3), we get

$$\begin{array}{rl} 1000x = & 134.343434 \ldots \\ 10x = & 1.343434 \ldots \\ (-) & \\ \hline 990x = & 133.00000 \end{array}$$

$$990x = 133$$

$$x = \frac{133}{990}$$

$\therefore x = \frac{133}{990}$, which is in $\frac{p}{q}$ form.

(vi) Let $x = 0.\overline{001}$

$$x = 0.001001 \ldots\ldots \rightarrow (1)$$

As there are three repeating digits after the decimal point, so multiplying both sides of (1) by 1000.

$$1000x = 1.001001 \ldots \rightarrow (2)$$

Subtracting (1) from (2), we get,

$$1000x = 1.001001\ldots$$
$$x = 0.001001\ldots$$
$$(-)$$
$$999x = 1$$
$$x = \frac{1}{999}$$

$\therefore\ x = 0.\overline{001} = \frac{1}{999}$, which is in $\frac{p}{q}$ form.

Solution (9)

(i) $\sqrt{23}$

Square root of 23 by long division method.

	4.79583
4	$23.\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}\,\overline{00}$
	16
87	700
	609
949	9100
	8541
9585	55900
	47925
95908	7,97,500
	7,67,264
959163	30,23600
	2877489
	146111

$\therefore\ \sqrt{23} = 4.79583$, which has non-terminating and non-repeating decimal Expansion.

So, it is an irrational number.

(ii) $\sqrt{225}$

Prime factorization of 225.

$$\begin{array}{r|l} 3 & 225 \\ \hline 3 & 75 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$225 = 3 \times 3 \times 5 \times 5.$$

$$225 = (3 \times 5)^2$$

$$\sqrt{225} = \sqrt{(3\times5)^2} = \left((3\times5)^{\cancel{2}}\right)^{\frac{1}{\cancel{2}}}$$

$$\therefore \sqrt{225} = 3 \times 5 = 15.$$

$\sqrt{225} = 15$; which is a rational number.

(iii) 0.3796

Decimal Expansion of 0.3796 is terminating.

So, $0.3796 = \frac{3796}{10000}$, which is in $\frac{p}{q}$ form.

$\therefore$ 0.3796 is a rational number.

(iv) $x = 7.478478 \ldots\ldots \rightarrow (1)$

As there are three repeating digits after the decimal point, So multiplying both sides of (1) by 1000.

$$1000x = 7478.478478 \ldots \rightarrow (2)$$

Subtracting (1) from (2), we get,

$$\begin{array}{rl} 1000x = & 7478.478478\ldots\ldots \\ x = & 7.478478\ldots \\ (-) & \\ \hline 999x = & 7471.0 \end{array}$$

$$x = \frac{7471}{999}$$

$\therefore$ $x = 7.478478\ldots\ldots = \frac{7471}{999}$, which is in '$\frac{p}{q}$' form.

So, $7.478478\ldots$ is a rational number.

(v) $1.101001000100001\ldots$

From the above decimal Expansion, we observed that after decimal point, number of zeros between two consecutive ones are increasing. So, it a non-terminating and non-repeating decimal Expantion.

$\therefore$ $1.101001000100001\ldots$ is an irrational number.

(vi) $345.0\overline{456}$

Let $x = 345.0456456\ldots \rightarrow (1)$.

Multiplying by 10 on both sides of Eq. (1)

$10x = 3450.456456\ldots \rightarrow (2)$

As there are three repeating digits after the decimal point, So multiplying both sides of (2) by 1000.

$10000x = 3450456.456456\ldots \rightarrow (3)$

(3) − (2) $\Rightarrow$

$$\begin{aligned} 10000x &= 3450456.456456\ldots \\ 10x &= 345.456456\ldots \\ (-) & \\ \hline 9990x &= 3450111.0 \end{aligned}$$

$\therefore\ 9990x = 3450111.$

$$x = \frac{3450111}{9990},$$

which is in the form $\frac{p}{q}$

So, $345.0\overline{456}$ is a rational number.

Solution (10) :-

(i) Decimal Expansion of $\frac{1}{3}$ and $\frac{1}{2}$.

$\therefore\ \frac{1}{3} = 0.333\ldots$

$= 0.\overline{3}$

$$\begin{array}{r|l} & 0.33 \\ \hline 3 & 10 \\ & 9 \\ \hline 3 & 10 \\ & 9 \\ \hline & 1 \end{array}$$

← remainder is repeating.

$\therefore\ \frac{1}{2} = 0.5$

$$\begin{array}{r|l} & 0.5 \\ \hline 2 & 10 \\ & 10 \\ \hline & 0 \end{array}$$

There are infinite rational numbers between $\frac{1}{3}(=0.\overline{3})$ and $\frac{1}{2}(=0.5)$.

One among them is $0.4040040004\ldots$

(ii) $\frac{-2}{5}$ and $\frac{1}{2}$.

Decimal Expansion of $\frac{-2}{5}$ and $\frac{1}{2}$.

$\therefore -\frac{2}{5} = -0.4$

$$\begin{array}{r|l} & 0.4 \\ \hline 5 & 20 \\ & 20 \\ \hline & 0 \end{array}$$

$\therefore \frac{1}{2} = 0.5$

$$\begin{array}{r|l} & 0.5 \\ \hline 2 & 10 \\ & 10 \\ \hline & 0 \end{array}$$

there are many irrational numbers between $\frac{-2}{5}$ and $\frac{1}{2}$. One among them is 0.1010010001....

(iii) 0 and 0.1

there are infinite irrational numbers between 0 and 0.1. One among them is

0.060060006000006 ....

Solution (11) :-

there are infinite irrational number between 2 and 3. Two among them are

2.010100100010000 1 . . . .

2.919119111911119 . . . .

Solution (12) :-

Decimal Expansion of $\frac{4}{9}$ and $\frac{7}{11}$

$\frac{4}{9} = 0.44\ldots\ldots$

$= 0.\overline{4}$

$$\begin{array}{r|l} & 0.44 \\ \hline 9 & 40 \\ & 36 \\ \hline & \;\;40 \\ & \;\;36 \\ \hline & \;\;\;\;4 \end{array}$$

Remainder is repeating

$\frac{7}{11} = 0.6363\ldots\ldots$

$= 0.\overline{63}$

$$\begin{array}{r|l} & 0.63 \\ \hline 11 & 70 \\ & 66 \\ \hline & 40 \\ & 33 \\ \hline & \;\;7 \end{array}$$

Remainder is repeating.

there are infinite rational number between $\frac{4}{9}(=0.\overline{4})$ and $\frac{7}{11}(=0.\overline{63})$.

Two among them are $0.404004000400004\ldots\ldots$,

$0.515115111511115\ldots\ldots$

Solution (13) :

Value of $\sqrt{2} = 1.414\ldots\ldots$

Value of $\sqrt{3} = 1.732\ldots\ldots$

there are many rational numbers between $\sqrt{2}$ and $\sqrt{3}$. One among them 1.6.

finding value of $\sqrt{2}$ and $\sqrt{3}$ by long division method.

$$
\begin{array}{r|l}
 & 1.414 \\
\hline
1 & 2.\overline{00}\,\overline{00}\,\overline{00} \\
 & 1 \\
\hline
24 & 100 \\
 & 96 \\
\hline
281 & 400 \\
 & 281 \\
\hline
2824 & 11900 \\
 & 11296 \\
\hline
 & 604
\end{array}
$$

$\therefore \sqrt{2} = 1.414\ldots\ldots$

$$
\begin{array}{r|l}
 & 1.732 \\
\hline
1 & 3.\overline{00}\,\overline{00}\,\overline{00} \\
 & 1 \\
\hline
27 & 200 \\
 & 189 \\
\hline
343 & 1100 \\
 & 1029 \\
\hline
3462 & 7100 \\
 & 6924 \\
\hline
 & 176
\end{array}
$$

$\therefore \sqrt{3} = 1.732\ldots\ldots$

Solution (14) :

$2\sqrt{3} = \sqrt{2^2} \times \sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{4 \times 3}$

$\therefore 2\sqrt{3} = \sqrt{12}$.

We have, $12 < 12.25 < 12.96 < 15$

$\Rightarrow \sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$.

$\sqrt{12} < \sqrt{(3.5)^2} < \sqrt{(3.6)^2} < \sqrt{15}$

$\sqrt{12} < 3.5 < 3.6 < \sqrt{15}$.

$\therefore$ 3.5 and 3.6 are two rational numbers between $\sqrt{12}$ and $\sqrt{15}$.

Solution (15) :

We have, $5 < 6 < 7$.

$\Rightarrow \sqrt{5} < \sqrt{6} < \sqrt{7}$

$\therefore \sqrt{6}$ is an irrational number between $\sqrt{5}$ and $\sqrt{7}$.

Solution (16)

We have, $3 < 5 < 6 < 7$

$\Rightarrow \sqrt{3} < \sqrt{5} < \sqrt{6} < \sqrt{7}$

$\therefore \sqrt{5}$ and $\sqrt{6}$ are two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$.

EXERCISE - 1.4

SOLUTION - 1

(i) $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Sol: $\sqrt{9\times5} - 3\sqrt{4\times5} + 4\sqrt{5}$

$= \sqrt{9}\sqrt{5} - 3\sqrt{4}\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 3\times2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5}$

$= (3-6+4)\sqrt{5}$

$= (7-6)\sqrt{5}$

$= \sqrt{5}$

(ii) $3\sqrt{3} + 2\sqrt{27} + \frac{7}{\sqrt{3}}$

Sol: $3\sqrt{3} + 2\sqrt{9\times3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 2\times\sqrt{9}\sqrt{3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 2\times3\sqrt{3} + \frac{7}{\sqrt{3}}$

$= 3\sqrt{3} + 6\sqrt{3} + \frac{7}{\sqrt{3}}$

$= (3+6)\sqrt{3} + \frac{7}{\sqrt{3}}$

$= 9\sqrt{3} + \frac{7}{\sqrt{3}}$

Multiplying and Dividing by "$\sqrt{3}$"

$$= 9\sqrt{3} + \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 9\sqrt{3} + \frac{7\sqrt{3}}{3}$$

By cross Multiplying

$$= \frac{3 \times 9\sqrt{3} + 7\sqrt{3}}{3}$$

$$= \frac{27\sqrt{3} + 7\sqrt{3}}{3}$$

$$= \frac{(27+7)\sqrt{3}}{3}$$

$$= \frac{34\sqrt{3}}{3}$$

(iii) $6\sqrt{5} \times 2\sqrt{5}$

Sol: $6 \times 2 \times \sqrt{5} \cdot \sqrt{5}$

$$= 12 \times (\sqrt{5})^2$$

$$= 12 \times 5$$

$$= 60$$

(iv) $8\sqrt{15} \div 2\sqrt{3}$

Sol: $\frac{8\sqrt{15}}{2\sqrt{3}} = \frac{\overset{4}{\cancel{8}}\sqrt{5}\sqrt{3}}{2\sqrt{3}}$

$$= 4\sqrt{5}$$

(v) $\frac{\sqrt{24}}{8} + \frac{\sqrt{54}}{9}$

Sol: $\frac{\sqrt{6\times4}}{8} + \frac{\sqrt{9\times6}}{9}$

$= \frac{\sqrt{6}\cdot\sqrt{4}}{8} + \frac{\sqrt{9}\cdot\sqrt{6}}{9}$

$= \frac{2\sqrt{6}}{8} + \frac{3\sqrt{6}}{9}$

$= \frac{1\cdot\sqrt{6}}{4} + \frac{1\cdot\sqrt{6}}{3}$

$= \sqrt{6}\left[\frac{1}{4} + \frac{1}{3}\right]$

$= \sqrt{6}\left[\frac{3+4}{12}\right]$ ∵ LCM of 4 and 3 is 12

$= \frac{7\sqrt{6}}{12}$

(vi) $\frac{3}{\sqrt{8}} + \frac{1}{\sqrt{2}}$

Sol: $\frac{3}{\sqrt{2\times4}} + \frac{1}{\sqrt{2}}$

$= \frac{3}{\sqrt{2}\cdot\sqrt{4}} + \frac{1}{\sqrt{2}}$

$= \frac{3}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}\left(\frac{3}{2} + \frac{1}{1}\right)$

$= \frac{1}{\sqrt{2}}\left(\frac{3+2}{2}\right)$ ∵ LCM of 2 and 1 is 2

$= \frac{1}{\sqrt{2}}\left(\frac{5}{2}\right)$

$= \frac{5}{2\sqrt{2}}$

Multiply and Divide by "$\sqrt{2}$"

$= \frac{5}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{5\sqrt{2}}{2\cdot\sqrt{2}\times\sqrt{2}}$

$= \frac{5\sqrt{2}}{2(\sqrt{2})^2} = \frac{5\sqrt{2}}{2\cdot 2}$

$= \frac{5\sqrt{2}}{4}$

SOLUTION - 2

(i) $(5+\sqrt{7})(2+\sqrt{5})$

Sol: $5\times 2 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{7}\cdot\sqrt{5}$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{7\times 5}$

$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(ii) $(5+\sqrt{5})(5-\sqrt{5})$

Sol: $(5)^2 - (\sqrt{5})^2$

$= 25 - 5$

$= 20$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Sol: $(\sqrt{5})^2+(\sqrt{2})^2+2\cdot\sqrt{5}\cdot\sqrt{2}$

$= 5+2+2\sqrt{5\times2}$

$= 5+2+2\sqrt{10}$

$= 7+2\sqrt{10}$

(iv) $(\sqrt{3}-\sqrt{7})^2$

Sol: $(\sqrt{3})^2+(\sqrt{7})^2-2\cdot\sqrt{3}\cdot\sqrt{7}$

$= 3+7-2\sqrt{3\times7}$

$= 10-2\sqrt{21}$

(v) $(\sqrt{2}+\sqrt{3})(\sqrt{5}+\sqrt{7})$

Sol: $\sqrt{2}\cdot\sqrt{5}+\sqrt{2}\cdot\sqrt{7}+\sqrt{3}\cdot\sqrt{5}+\sqrt{3}\cdot\sqrt{7}$

$= \sqrt{2\times5}+\sqrt{2\times7}+\sqrt{3\times5}+\sqrt{3\times7}$

$= \sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}$

(vi) $(4+\sqrt{5})(\sqrt{3}-\sqrt{7})$

Sol: $4\sqrt{3}-4\sqrt{7}+\sqrt{5}\cdot\sqrt{3}-\sqrt{5}\cdot\sqrt{7}$

$= 4\sqrt{3}-4\sqrt{7}+\sqrt{5\times3}-\sqrt{5\times7}$

$= 4\sqrt{3}-4\sqrt{7}+\sqrt{15}-\sqrt{35}$

SOLUTION - 3

(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Sol: $\sqrt{4\times 2} + \sqrt{25\times 2} + \sqrt{36\times 2} + \sqrt{49\times 2}$

$= \sqrt{4}\cdot\sqrt{2} + \sqrt{25}\cdot\sqrt{2} + \sqrt{36}\cdot\sqrt{2} + \sqrt{49}\cdot\sqrt{2}$

$= 2\sqrt{2} + 5\sqrt{2} + 6\sqrt{2} + 7\sqrt{2}$

$= (2+5+6+7)\sqrt{2}$

$= 20 \times \sqrt{2}$

$= 20 \times 1.414$

$= 28.28$

(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$

Sol: $3\sqrt{16\times 2} - 2\sqrt{25\times 2} + 4\sqrt{64\times 2} - 20\sqrt{9\times 2}$

$= 3\times 4\sqrt{2} - 2\times 5\sqrt{2} + 4\times 8\sqrt{2} - 20\times 3\sqrt{2}$

$= 12\sqrt{2} - 10\sqrt{2} + 32\sqrt{2} - 60\sqrt{2}$

$= (12 - 10 + 32 - 60)\sqrt{2}$

$= -26 \times \sqrt{2}$

$= -26 \times 1.414$

$= -36.764$

SOLUTION - 4

(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$

Sol: $\sqrt{9 \times 3} + \sqrt{25 \times 3} + \sqrt{36 \times 3} - \sqrt{81 \times 3}$

$= 3\sqrt{3} + 5\sqrt{3} + 6\sqrt{3} - 9\sqrt{3}$

$= (3+5+6-9)\sqrt{3}$

$= (14-9)\sqrt{3}$

$= 5 \times \sqrt{3}$

$= 5 \times 1.732$

$= 8.66$

(ii) $\cancel{3\sqrt{3/2}}$ $5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$

Sol: $5\sqrt{4 \times 3} - 3\sqrt{16 \times 3} + 6\sqrt{25 \times 3} + 7\sqrt{36 \times 3}$

$= 5 \times 2\sqrt{3} - 3 \times 4\sqrt{3} + 6 \times 5\sqrt{3} + 7 \times 6\sqrt{3}$

$= 10\sqrt{3} - 12\sqrt{3} + 30\sqrt{3} + 42\sqrt{3}$

$= (10 - 12 + 30 + 42)\sqrt{3}$

$= (82 - 12)\sqrt{3}$

$= 70 \times \sqrt{3}$

$= 70 \times 1.732$

$= 121.24$

SOLUTION – 5

(i) $\sqrt{\frac{4}{9}}$, $-\frac{3}{70}$, $\sqrt{\frac{7}{25}}$, $\sqrt{\frac{16}{5}}$

Sol: $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}}$

$= \frac{2}{3}$. It is in the form of $\frac{p}{q}$ and p, q are Integers

Therefore $\sqrt{\frac{4}{9}}$ is a rational numbe.

$\Rightarrow$ $-\frac{3}{70}$ is a rational number

$\Rightarrow$ $\sqrt{\frac{7}{25}} = \frac{\sqrt{7}}{\sqrt{25}}$

$= \frac{\sqrt{7}}{5}$. Since $\sqrt{7}$ is not an Integer

Therefore, $\sqrt{\frac{7}{25}}$ is an Irrational number

$\Rightarrow$ $\sqrt{\frac{16}{5}} = \frac{\sqrt{16}}{\sqrt{5}}$

$= \frac{4}{\sqrt{5}}$ Since $\sqrt{5}$ is not an Integer.

Therefore, $\sqrt{\frac{16}{5}}$ is an Irrational number

(ii) $-\sqrt{\frac{2}{49}}$, $\frac{3}{200}$, $\sqrt{\frac{25}{3}}$, $-\sqrt{\frac{49}{16}}$

Sol: $-\sqrt{\frac{2}{49}} = -\frac{\sqrt{2}}{\sqrt{49}}$

$= -\frac{\sqrt{2}}{7}$. Since $\sqrt{2}$ is not an Integer

Therefore, $-\sqrt{\frac{2}{49}}$ is an Irrational number

$\Rightarrow$ $\frac{3}{200}$. It is in the form of $\frac{p}{q}$ and p, q are Integers . So, It is a rational number

$\Rightarrow$ $\sqrt{\frac{25}{3}} = \frac{\sqrt{25}}{\sqrt{3}}$

$= \frac{5}{\sqrt{3}}$ since $\sqrt{3}$ is not an Integer

Therefore, $\sqrt{\frac{25}{3}}$ is an Irrational number

$\Rightarrow$ $-\sqrt{\frac{49}{16}} = -\frac{\sqrt{49}}{\sqrt{16}}$

$= -\frac{7}{4}$

Therefore, $-\sqrt{\frac{49}{16}}$ is a rational number

SOLUTION – 6

(i) $-3\sqrt{2}$

Sol: Since $\sqrt{2}$ is an irrational number

Therefore, $-3\sqrt{2}$ will change into non-terminating non-recurring decimal

(ii) $\sqrt{\frac{256}{81}}$

Sol:- $\frac{\sqrt{256}}{\sqrt{81}} = \frac{16}{9}$

$= 1.77777\overline{7}$

Therefore, $\sqrt{\frac{256}{81}}$ will not change into non-terminating non-recurring decimal

(iii) $\sqrt{27 \times 16}$

Sol:- $\sqrt{27} \times \sqrt{16} = \sqrt{9 \times 3} \times 4$

$= 4 \times 3\sqrt{3}$

$= 12\sqrt{3}$

$\because \sqrt{3}$ is an irrational number

Therefore, $\sqrt{27 \times 16}$ will change into non-terminating non-recurring decimal

(iv) $\sqrt{\frac{5}{36}}$

Sol:- $\frac{\sqrt{5}}{\sqrt{36}} = \frac{\sqrt{5}}{6}$

$\because \sqrt{5}$ is an irrational number

Therefore, $\sqrt{\frac{5}{36}}$ will change into non-terminating non-recurring decimal

SOLUTION - 7

(i) $3 - \sqrt{\frac{7}{25}}$

Sol: $3 - \frac{\sqrt{7}}{\sqrt{25}}$

$= 3 - \frac{\sqrt{7}}{5}$

$= \frac{15 - \sqrt{7}}{5}$ $\because \sqrt{7}$ is an irrational number

Therefore, $3 - \sqrt{\frac{7}{25}}$ is also an irrational number

(ii) $-\frac{2}{3} + \sqrt[3]{2}$

Sol: Since $\sqrt[3]{2}$ is an irrational number.

Therefore, $-\frac{2}{3} + \sqrt[3]{2}$ is also an irrational number

(NOTE: Sum of rational and irrational number is irrational)

(iii) $\frac{3}{\sqrt{3}}$

Sol: $\frac{\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = \sqrt{3}$

Since $\sqrt{3}$ is an irrational number

Therefore, $\frac{3}{\sqrt{3}}$ is also an irrational number.

(iv) $-\frac{2}{7}\sqrt[3]{5}$

Sol: Since $\sqrt[3]{5}$ is an Irrational number

Therefore, $-\frac{2}{7}\sqrt[3]{5}$ is also an Irrational number.

(NOTE: Product of rational and Irrational number is Irrational)

(v) $(2-\sqrt{3})(2+\sqrt{3})$

Sol: $2\times2 + 2\sqrt{3} - 2\sqrt{3} - (\sqrt{3})^2$

$= 4 - 3$

$= 1$

Therefore, $(2-\sqrt{3})(2+\sqrt{3})$ is a rational number

(vi) $(3+\sqrt{5})^2$

Sol: $(3)^2 + (\sqrt{5})^2 + 2\times3\times\sqrt{5}$

$= 9 + 5 + 6\sqrt{5}$

$= 14 + 6\sqrt{5}$

Since $\sqrt{5}$ is an Irrational number

$(3+\sqrt{5})^2$ is also an Irrational number

(VII) $\left(\frac{2}{5}\sqrt{7}\right)^2$

Sol: $\left(\frac{2}{5}\right)^2 \cdot (\sqrt{7})^2$

$= \frac{4}{25} \times 7$

$= \frac{28}{25}$

Therefore, $\left(\frac{2}{5}\sqrt{7}\right)^2$ is a rational number

(VIII) $(3-\sqrt{6})^2$

Sol: $(3)^2 + (\sqrt{6})^2 - 2 \times 3 \times \sqrt{6}$

$= 9 + 6 - 6\sqrt{6}$

$= 15 - 6\sqrt{6}$

Since $\sqrt{6}$ is an Irrational number

$(3-\sqrt{6})^2$ is also an Irrational number

SOLUTION -8 :

(i) $\sqrt[3]{2}$

Sol: Suppose that $\sqrt[3]{2} = \frac{p}{q}$, Where p, q are integers, $q > 0$, p and q have no common factors (except 1)

$$2 = \left[\frac{p}{q}\right]^3$$

$$p^3 = 2q^3 \longrightarrow (1)$$

As 2 divides $2q^3$ $\Rightarrow$ 2 divides p^3

$\Rightarrow$ 2 divides p

let $p = 2K$, Where K is an Integer

Substituting this value of 'p' in ①, We get

$$(2K)^3 = 2q^3$$

$$8K^3 = 2q^3$$

$$4K^3 = q^3$$

As 2 divides $4K^3$ $\Rightarrow$ 2 divides q^3

$\Rightarrow$ 2 divides q

Thus p and q have a common factor "2".

This contradicts that p and q have no common factor (except 1)

Therefore, $\sqrt[3]{2}$ is an Irrational number

(ii) $\sqrt[3]{3}$

Sol: Suppose that $\sqrt[3]{3} = \frac{p}{q}$, Where p, q are Integers $q > 0$, p and q have no common factors (except 1)

$$3 = \left(\frac{p}{q}\right)^3$$

$$p^3 = 3q^3 \longrightarrow ①$$

As 3 divides $3q^3$ $\Rightarrow$ 3 divides p^3

$\Rightarrow$ 3 divides p

Let $p = 3K$, Where K is an Integer

Substituting this value of 'p' in ①, We get

$$(3K)^3 = 3q^3$$

$$27K^3 = 3q^3$$

$$9K^3 = q^3$$

As 3 divides $9K^3 \Rightarrow$ 3 divides q^3

$\Rightarrow$ 3 divide q

Thus p and q have a Common factor "3"

This contradicts that p and q have no common factor (except 1)

Therefore, $\sqrt[3]{3}$ is an Irrational number.

(iii) $\sqrt[4]{5}$

Sol: Suppose that $\sqrt[4]{5} = \frac{p}{q}$, Where p, q are Integers $q > 0$, p and q have no common factors (except 1)

$$5 = \left(\frac{p}{q}\right)^4$$

$$p^4 = 5q^4 \longrightarrow ①$$

As 5 divides $5q^4 \Rightarrow$ 5 divides p^4

$\Rightarrow$ 5 divides p

Let $p = 5K$, Where K is an Integer

Substituting this value of 'p' in ①, We get

$$(5K)^4 = 5q^4$$

$$625K^4 = 5q^4$$

$$125K^4 = q^4$$

As 5 divides $125K^4 \Rightarrow$ 5 divides q^4

$\Rightarrow$ 5 divides q

Thus p and q have a common factor "5"

This contradicts that p and q have no common factor (except 1)

Therefore, $\sqrt[4]{5}$ is an irrational number.

SOLUTION-9

(i) $2\sqrt{3}$, $\frac{3}{\sqrt{2}}$, $-\sqrt{7}$, $\sqrt{15}$

Sol: $\sqrt{4\times3} = \sqrt{12}$

$$\frac{3}{\sqrt{2}} = \sqrt{\frac{9}{2}}$$

$$= \sqrt{4.5}$$

$\therefore$ $\sqrt{12}$, $\sqrt{4.5}$, $-\sqrt{7}$, $\sqrt{15}$

$$-\sqrt{7} < \sqrt{4.5} < \sqrt{12} < \sqrt{15}$$

The greatest real number is $\sqrt{15}$

The smallest real number is $-\sqrt{7}$

(ii) $-3\sqrt{2}$, $\frac{9}{\sqrt{5}}$, -4 , $\frac{4}{3}\sqrt{5}$, $\frac{3}{2}\sqrt{3}$

Sol: $-3\sqrt{2} = -\sqrt{9\times 2}$

$= -\sqrt{18}$

$\frac{9}{\sqrt{5}} = \sqrt{\frac{81}{5}}$

$= \sqrt{16.2}$

$-4 = -\sqrt{16}$

$\frac{4}{3}\sqrt{5} = \sqrt{\frac{16\times 5}{9}}$

$= \sqrt{\frac{80}{9}}$

$= \sqrt{8.89}$

$\frac{3}{2}\sqrt{3} = \sqrt{\frac{9\times 3}{4}}$

$= \sqrt{\frac{27}{4}}$

$= \sqrt{6.75}$

$\therefore -\sqrt{18}$, $\sqrt{16.2}$, $\sqrt{8.89}$, $-\sqrt{16}$, $\sqrt{6.75}$

$-3\sqrt{2} < -4 < \frac{3}{2}\sqrt{3} < \frac{4}{3}\sqrt{5} < \frac{9}{\sqrt{5}}$

The greatest real number is $\frac{9}{\sqrt{5}}$

The smallest real number is $-3\sqrt{2}$

SOLUTION - 10

(i) $3\sqrt{2}$, $2\sqrt{3}$, $\sqrt{15}$, 4

Sol: Write all the numbers as square roots under one radical

$3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{18}$

$2\sqrt{3} = \sqrt{4} \times \sqrt{3} = \sqrt{12}$

$\sqrt{15} = \sqrt{15}$

$4 = \sqrt{16}$

Since $12 < 15 < 16 < 18$

$\Rightarrow \sqrt{12} < \sqrt{15} < \sqrt{16} < \sqrt{18}$

$\Rightarrow 2\sqrt{3} < \sqrt{15} < 4 < 3\sqrt{2}$

Hence, the given numbers in ascending orders are

$2\sqrt{3}$, $\sqrt{15}$, 4 , $3\sqrt{2}$

(ii) $3\sqrt{2}$, $2\sqrt{8}$, 4 , $\sqrt{50}$, $4\sqrt{3}$

Sol: Write all the numbers as square roots under one radical

$3\sqrt{2} = \sqrt{9} \times \sqrt{2} = \sqrt{18}$

$2\sqrt{8} = \sqrt{4} \times \sqrt{8} = \sqrt{32}$

$4 = \sqrt{16}$

$\sqrt{50} = \sqrt{50}$

$4\sqrt{3} = \sqrt{16} \times \sqrt{3} = \sqrt{48}$

Since $16 < 18 < 32 < 48 < 50$

$$\Rightarrow \sqrt{16} < \sqrt{18} < \sqrt{32} < \sqrt{48} < \sqrt{50}$$

$$\Rightarrow 4 < 3\sqrt{2} < 2\sqrt{8} < 4\sqrt{3} < \sqrt{50}$$

Hence, the given numbers in ascending orders are

$4, 3\sqrt{2}, 2\sqrt{8}, 4\sqrt{3}, \sqrt{50}$

SOLUTION - 11

(i) $\frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 4\sqrt{3}, 3\sqrt{\frac{6}{5}}$

Sol: Write all the numbers as square roots under one radical

$$\frac{9}{\sqrt{2}} = \sqrt{\frac{81}{2}} = \sqrt{40.5}$$

$$\frac{3}{2}\sqrt{5} = \sqrt{\frac{9}{4}} \times \sqrt{5} = \sqrt{\frac{45}{4}} = \sqrt{11.25}$$

$$4\sqrt{3} = \sqrt{16} \times \sqrt{3} = \sqrt{48}$$

$$3\sqrt{\frac{6}{5}} = \sqrt{9} \times \sqrt{\frac{6}{5}} = \sqrt{\frac{54}{5}} = \sqrt{10.8}$$

Since $48 > 40.5 > 11.25 > 10.8$

$$\Rightarrow \sqrt{48} > \sqrt{40.5} > \sqrt{11.25} > \sqrt{10.8}$$

$$\Rightarrow 4\sqrt{3} > \frac{9}{\sqrt{2}} > \frac{3}{2}\sqrt{5} > 3\sqrt{\frac{6}{5}}$$

Hence, the given numbers in descending orders are $4\sqrt{3}, \frac{9}{\sqrt{2}}, \frac{3}{2}\sqrt{5}, 3\sqrt{\frac{6}{5}}$

(ii) $\frac{5}{\sqrt{3}}, \frac{7}{3}\sqrt{2}, -\sqrt{3}, 3\sqrt{5}, 2\sqrt{7}$

Sol: Write all the numbers as square roots under one radical

$$\frac{5}{\sqrt{3}} = \sqrt{\frac{25}{3}} = \sqrt{8.33}$$

$$\frac{7}{3}\sqrt{2} = \sqrt{\frac{49}{9}} \times \sqrt{2} = \sqrt{\frac{98}{9}} = \sqrt{10.89}$$

$$-\sqrt{3} = -\sqrt{3}$$

$$3\sqrt{5} = \sqrt{9} \times \sqrt{5} = \sqrt{45}$$

$$2\sqrt{7} = \sqrt{4} \times \sqrt{7} = \sqrt{28}$$

Since $45 > 28 > 10.89 > 8.33 > -3$

$$\Rightarrow \sqrt{45} > \sqrt{28} > \sqrt{10.89} > \sqrt{8.33} > -\sqrt{3}$$

$$\Rightarrow 3\sqrt{5} > 2\sqrt{7} > \frac{7}{3}\sqrt{2} > \frac{5}{\sqrt{3}} > -\sqrt{3}$$

Hence, the given numbers in descending orders are $3\sqrt{5}, 2\sqrt{7}, \frac{7}{3}\sqrt{2}, \frac{5}{\sqrt{3}}, -\sqrt{3}$

SOLUTION - 12

(i) $\sqrt[3]{2}$, $\sqrt{3}$, $\sqrt[6]{5}$

<u>Sol</u>: L.C.M. of 2, 3 and 6 is 6

$$\sqrt[3]{2} = 2^{\frac{1}{3}} = (2^2)^{\frac{1}{6}} = (4)^{\frac{1}{6}}$$

$$\sqrt{3} = 3^{\frac{1}{2}} = (3^3)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$$

$$\sqrt[6]{5} = 5^{\frac{1}{6}} = (5^1)^{\frac{1}{6}} = (5)^{\frac{1}{6}}$$

Since $4 < 5 < 27$

$$\Rightarrow (4)^{\frac{1}{6}} < (5)^{\frac{1}{6}} < (27)^{\frac{1}{6}}$$

$$\Rightarrow \sqrt[3]{2} < \sqrt[6]{5} < \sqrt{3}$$

Hence, the given numbers in ascending orders are $\sqrt[3]{2}$, $\sqrt[6]{5}$, $\sqrt{3}$

EXERCISE - 1.5

SOLUTION - 1

(i) $\frac{3}{4\sqrt{5}}$

Sol: $\frac{3}{4\sqrt{5}} \times \frac{4\sqrt{5}}{4\sqrt{5}} = \frac{12\sqrt{5}}{16\cdot(\sqrt{5})^2}$

$= \frac{12\sqrt{5}}{16 \times 5}$

$= \frac{3\sqrt{5}}{20}$

(ii) $\frac{5\sqrt{7}}{\sqrt{3}}$

Sol: $\frac{5\sqrt{7}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{7 \times 3}}{(\sqrt{3})^2}$

$= \frac{5\sqrt{21}}{3}$

(iii) $\frac{3}{4-\sqrt{7}}$

Sol: $\frac{3}{4-\sqrt{7}} \times \frac{4+\sqrt{7}}{4+\sqrt{7}} = \frac{3(4+\sqrt{7})}{(4-\sqrt{7})(4+\sqrt{7})}$

$= \frac{12 + 3\sqrt{7}}{16-(\sqrt{7})^2}$

$= \frac{12 + 3\sqrt{7}}{16-7}$

$= \frac{12 + 3\sqrt{7}}{9}$

$$= \frac{3(4+\sqrt{7})}{9}$$

$$= \frac{4+\sqrt{7}}{3}$$

(iv) $\frac{17}{3\sqrt{2}+1}$

Sol: $\frac{17}{3\sqrt{2}+1} \times \frac{3\sqrt{2}-1}{3\sqrt{2}-1} = \frac{17(3\sqrt{2}-1)}{(3\sqrt{2}+1)(3\sqrt{2}-1)}$

$$= \frac{51\sqrt{2}-17}{(3\sqrt{2})^2-(1)^2}$$

$$= \frac{17(3\sqrt{2}-1)}{9\times 2-1}$$

$$= \frac{17(3\sqrt{2}-1)}{18-1}$$

$$= \frac{17(3\sqrt{2}-1)}{17}$$

$$= 3\sqrt{2}-1$$

(v) $\frac{16}{\sqrt{41}-5}$

Sol: $\frac{16}{\sqrt{41}-5} \times \frac{\sqrt{41}+5}{\sqrt{41}+5} = \frac{16(\sqrt{41}+5)}{(\sqrt{41})^2-(5)^2}$

$$= \frac{16(\sqrt{41}+5)}{41-25}$$

$$= \frac{16(\sqrt{41}+5)}{16}$$

$$= \sqrt{41}+5$$

(vi) $\frac{1}{\sqrt{7}-\sqrt{6}}$

<u>Sol</u>: $\frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} = \frac{\sqrt{7}+\sqrt{6}}{(\sqrt{7})^2-(\sqrt{6})^2}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$

$= \sqrt{7}+\sqrt{6}$

(vii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

<u>Sol</u>: $\frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$

$= \frac{\sqrt{5}-\sqrt{2}}{3}$

(viii) $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

<u>Sol</u>: $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \times \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{(\sqrt{2})^2+(\sqrt{3})^2+2\cdot\sqrt{2}\cdot\sqrt{3}}{2-3}$

$= \frac{2+3+2\sqrt{6}}{-1}$

$= -(5+2\sqrt{6})$

$= -5-2\sqrt{6}$

SOLUTION-2

(i) $\dfrac{7+3\sqrt{5}}{7-3\sqrt{5}}$

Sol:- $\dfrac{7+3\sqrt{5}}{7-3\sqrt{5}} \times \dfrac{7+3\sqrt{5}}{7+3\sqrt{5}} = \dfrac{(7+3\sqrt{5})^2}{(7)^2-(3\sqrt{5})^2}$

$= \dfrac{(7)^2+(3\sqrt{5})^2+2\times7\times3\sqrt{5}}{49-9\times5}$

$= \dfrac{49+45+14\times3\sqrt{5}}{49-45}$

$= \dfrac{94+42\sqrt{5}}{4}$

$= \dfrac{2(47+21\sqrt{5})}{4}$

$= \dfrac{47+21\sqrt{5}}{2}$

(ii) $\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}}$

Sol: $\dfrac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \dfrac{3-2\sqrt{2}}{3-2\sqrt{2}} = \dfrac{(3-2\sqrt{2})^2}{(3)^2-(2\sqrt{2})^2}$

$= \dfrac{(3)^2+(2\sqrt{2})^2-2\times3\times2\sqrt{2}}{9-4\times2}$

$= \dfrac{9+8-12\sqrt{2}}{9-8}$

$= \dfrac{17-12\sqrt{2}}{1}$

$= 17-12\sqrt{2}$

(iii) $\dfrac{5-3\sqrt{14}}{7+2\sqrt{14}}$

Sol: $\dfrac{5-3\sqrt{14}}{7+2\sqrt{14}} \times \dfrac{7-2\sqrt{14}}{7-2\sqrt{14}}$

$$= \frac{5\times 7 - 5\times 2\sqrt{14} - 7\times 3\sqrt{14} + 2\times 3\times\sqrt{14}\cdot\sqrt{14}}{(7)^2 - (2\sqrt{14})^2}$$

$$= \frac{35 - 10\sqrt{14} - 21\sqrt{14} + 6\times 14}{49 - 4\times 14}$$

$$= \frac{35 - 31\sqrt{14} + 84}{49-56}$$

$$= \frac{119 - 31\sqrt{14}}{-7}$$

$$= \frac{31\sqrt{14} - 119}{7}$$

SOLUTION - 3

(i) $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \dfrac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \dfrac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

Sol: $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \dfrac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$

$$= \frac{7\sqrt{30} - 7\times 3}{(\sqrt{10})^2 - (\sqrt{3})^2}$$

$$= \frac{7\sqrt{30} - 21}{10-3}$$

$$= \frac{7(\sqrt{30}-3)}{7} = \sqrt{30} - 3$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

$$= \frac{2\sqrt{30} - 2\times 5}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{2\sqrt{30} - 10}{6-5}$$

$$= 2\sqrt{30} - 10$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$= \frac{3\sqrt{30} - 9\times 2}{(\sqrt{15})^2 - (3\sqrt{2})^2}$$

$$= \frac{3\sqrt{30} - 18}{15 - 18}$$

$$= \frac{3(\sqrt{30} - 6)}{-2}$$

$$= -\sqrt{30} + 6$$

$$\therefore \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$= \sqrt{30} - 3 - (2\sqrt{30} - 10) - (-\sqrt{30} + 6)$$

$$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6$$

$$= 10 - 9 + \cancel{2\sqrt{30}} - \cancel{2\sqrt{30}}$$

$$= 1$$

SOLUTION - 4

(i) $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}} = -\dfrac{19}{11} + a\sqrt{5}$

Sol: $\dfrac{3-\sqrt{5}}{3+2\sqrt{5}} \times \dfrac{3-2\sqrt{5}}{3-2\sqrt{5}} = \dfrac{(3-\sqrt{5})(3-2\sqrt{5})}{(3)^2-(2\sqrt{5})^2}$

$$= \frac{9-6\sqrt{5}-3\sqrt{5}+2(5)}{9-4(5)}$$

$$= \frac{19-9\sqrt{5}}{9-20}$$

$$= \frac{+19-9\sqrt{5}}{-11}$$

$$= -\frac{19}{11} + \frac{9}{11}\sqrt{5}$$

$\therefore -\dfrac{19}{11} + a\sqrt{5} = -\dfrac{14}{11} + \dfrac{9}{11}\sqrt{5}$

$\Rightarrow a = \dfrac{9}{11}$

(ii) $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = a - b\sqrt{6}$

Sol: $\dfrac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \times \dfrac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}} = \dfrac{(\sqrt{2}+\sqrt{3})(3\sqrt{2}+2\sqrt{3})}{(3\sqrt{2})^2-(2\sqrt{3})^2}$

$$= \frac{3(2)+2\sqrt{6}+3\sqrt{6}+2(3)}{9(2)-4(3)}$$

$$= \frac{6+5\sqrt{6}+6}{18-12}$$

$$= \frac{12 + 5\sqrt{6}}{6}$$

$$= 2 + \frac{5}{6}\sqrt{6}$$

$$= 2 - \left(-\frac{5}{6}\right)\sqrt{6}$$

$\therefore\ a - b\sqrt{6} = 2 - \left(-\frac{5}{6}\right)\sqrt{6}$

$\Rightarrow a = 2 \ ; \ b = -\frac{5}{6}$

(iii) $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = a + \frac{7}{11} b\sqrt{5}$

Sol. $\frac{7+\sqrt{5}}{7-\sqrt{5}} \times \frac{7+\sqrt{5}}{7+\sqrt{5}} = \frac{(7+\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2}$

$$= \frac{49 + (\sqrt{5})^2 + 2 \cdot 7 \cdot \sqrt{5}}{49 - 5}$$

$$= \frac{49 + 5 + 14\sqrt{5}}{44}$$

$\frac{7-\sqrt{5}}{7+\sqrt{5}} \times \frac{7-\sqrt{5}}{7-\sqrt{5}} = \frac{(7-\sqrt{5})^2}{(7)^2 - (\sqrt{5})^2}$

$$= \frac{49 + (\sqrt{5})^2 - 2 \times 7 \times \sqrt{5}}{49 - 5}$$

$$= \frac{49 + 5 - 14\sqrt{5}}{44}$$

$\therefore\ \frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = \frac{54 + 14\sqrt{5}}{44} - \frac{54 - 14\sqrt{5}}{44}$

$$= \frac{\cancel{54} + 14\sqrt{5} - \cancel{54} + 14\sqrt{5}}{44}$$

$$= \frac{7}{11} \cdot \frac{28\sqrt{5}}{\cancel{44}}$$

$$= \frac{7}{11} \times \sqrt{5}$$

$$\therefore \; a + \frac{7}{11} b\sqrt{5} = \frac{7}{11}\sqrt{5}$$

$$\Rightarrow a = 0 \; ; \; b = 1$$

SOLUTION - 5 :

(i) $$\frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = p + q\sqrt{5}$$

Sol: $$\frac{7+3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{(7+3\sqrt{5})(3-\sqrt{5})}{(3)^2 - (\sqrt{5})^2}$$

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 3(5)}{9-5}$$

$$= \frac{21 + 2\sqrt{5} - 15}{4}$$

$$= \frac{6+2\sqrt{5}}{4}$$

$$\frac{7-3\sqrt{5}}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{(7-3\sqrt{5})(3+\sqrt{5})}{(3)^2 - (\sqrt{5})^2}$$

$$= \frac{21 + 7\sqrt{5} - 9\sqrt{5} - 3(5)}{9-5}$$

$$= \frac{21 - 2\sqrt{5} - 15}{4}$$

$$= \frac{6-2\sqrt{5}}{4}$$

$$\therefore \frac{7+3\sqrt{5}}{3+\sqrt{5}} - \frac{7-3\sqrt{5}}{3-\sqrt{5}} = \frac{6+2\sqrt{5}}{4} - \frac{6-2\sqrt{5}}{4}$$

$$= \frac{6+2\sqrt{5}-6+2\sqrt{5}}{4}$$

$$= \frac{4\sqrt{5}}{4}$$

$$= \sqrt{5}$$

$$\therefore P + q\sqrt{5} = \sqrt{5}$$

$$\Rightarrow P = 0 \; ; \; q = 1$$

SOLUTION -6 :

(i) $\frac{\sqrt{2}}{2+\sqrt{2}}$

Sol: $\frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{\sqrt{2}(2-\sqrt{2})}{(2)^2-(\sqrt{2})^2}$

$$= \frac{2\sqrt{2}-2}{4-2}$$

$$= \frac{\cancel{2}(\sqrt{2}-1)}{\cancel{2}}$$

$$= \sqrt{2}-1$$

$$= 1.414-1$$

$$= 0.414$$

(ii) $\frac{1}{\sqrt{3}+\sqrt{2}}$

Sol: $\frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3}-\sqrt{2}$

$= 1.732 - 1.414$

$= 0.318$

SOLUTION - 7:

(i) $a = 2+\sqrt{3}$

Sol:- $\frac{1}{a} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$

$= \frac{2-\sqrt{3}}{4-3}$

$= 2-\sqrt{3}$

$\therefore a-\frac{1}{a} = 2+\sqrt{3} - (2-\sqrt{3})$

$= \cancel{2}+\sqrt{3} - \cancel{2}+\sqrt{3}$

$= 2\sqrt{3}$

SOLUTION – 8

(i) $x = 1-\sqrt{2}$

Sol: Given $x = 1-\sqrt{2}$

$$\therefore \frac{1}{x} = \frac{1}{1-\sqrt{2}} = \frac{1}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}}$$

$$= \frac{1+\sqrt{2}}{(1)^2-(\sqrt{2})^2}$$

$$= \frac{1+\sqrt{2}}{1-2}$$

$$= -(1+\sqrt{2})$$

$$\therefore \left(x-\frac{1}{x}\right)^4 = \left(1-\sqrt{2}-(-1-\sqrt{2})\right)^4$$

$$= (1-\sqrt{2}+1+\sqrt{2})^4$$

$$= 2^4$$

$$= 16$$

SOLUTION – 9

(i) $x = 5-2\sqrt{6}$

Sol: Given $x = 5-2\sqrt{6}$

$$\therefore \frac{1}{x} = \frac{1}{5-2\sqrt{6}} = \frac{1}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$$

$$= \frac{5+2\sqrt{6}}{(5)^2-(2\sqrt{6})^2}$$

$$= \frac{5+2\sqrt{6}}{25-24}$$

$$= 5+2\sqrt{6}$$

$$\therefore\ x+\frac{1}{x} = (5-2\sqrt{6})+(5+2\sqrt{6})$$

$$= 10$$

We Know that $\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$

$$\Rightarrow x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2-2$$

$$= (10)^2-2$$

$$= 100-2$$

$$= 98$$

SOLUTION - 10

(i) $p = \frac{2-\sqrt{5}}{2+\sqrt{5}}$; $q = \frac{2+\sqrt{5}}{2-\sqrt{5}}$

Sol: $p+q = \frac{2-\sqrt{5}}{2+\sqrt{5}} + \frac{2+\sqrt{5}}{2-\sqrt{5}}$

$$= \frac{(2-\sqrt{5})^2+(2+\sqrt{5})^2}{(2)^2-(\sqrt{5})^2}$$

$$= \frac{(4+5-4\sqrt{5})+(4+5+4\sqrt{5})}{4-5}$$

$$= \frac{18}{-1}$$

$$\therefore\ p+q = -18$$

(ii) $p - q = \dfrac{2-\sqrt{5}}{2+\sqrt{5}} - \dfrac{2+\sqrt{5}}{2-\sqrt{5}}$

$$= \frac{(2-\sqrt{5})^2 - (2+\sqrt{5})^2}{(2)^2 - (\sqrt{5})^2}$$

$$= \frac{(4+5-4\sqrt{5}) - (4+5+4\sqrt{5})}{4-5}$$

$$= \frac{9-4\sqrt{5}-9-4\sqrt{5}}{-1}$$

$$= \frac{-8\sqrt{5}}{-1}$$

$$= 8\sqrt{5}$$

(iii) p^2+q^2

Sol: We Know that

$$(p+q)^2 = p^2+q^2+2pq$$

$$\therefore pq = \frac{2-\sqrt{5}}{2+\sqrt{5}} \times \frac{2+\sqrt{5}}{2-\sqrt{5}} = 1$$

$$\therefore p+q = -18$$

$$\Rightarrow p^2+q^2 = (p+q)^2 - 2pq$$

$$= (-18)^2 - 2\times 1$$

$$= 324-2$$

$$= 322$$

(iv) $p^2 - q^2$

Sol:- $\therefore\ p^2 - q^2 = (p+q)(p-q)$

$= (-18)(8\sqrt{5})$

$= -144\sqrt{5}$