Ex 3.1

Question 1.

Which of the following natural numbers are perfect squares? Give reasons in support of your answer.

(i) 729

(ii) 5488

(iii) 1024

(iv) 243

Solution:

(i) 729

 $\frac{3|729}{3|243}$ $\frac{3|81}{3|27}$ $\frac{3|9}{3|3}$ $= 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

729 is the product of pairs of equal prime factors.

729 is a perfect square.

(ii) 5488

2 5488	
2 2744	
2 1372	
2 686	
7 343	
7 49	
7 7	
1	

 $= 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7$

After pairing the same prime factors,

we see that one factor 7 is left unpaired.

So, 5488 is not a perfect square.

(iii) 1024

 $\frac{2|1024}{2|512} \\
\frac{2|512}{2|256} \\
\frac{2|128}{2|64} \\
\frac{2|64}{2|32} \\
\frac{2|16}{2|8} \\
\frac{2|4}{2|2} \\
1$

After pairing the same prime factors, we see that there is no factor left.

So, 1024 is a perfect square.

(iv) 243

 $\frac{3|243}{3|81} \\ \frac{3|27}{\frac{3|9}{\frac{3|3}{1}}} \\ = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

After pairing the same prime factors.

We see that factor 3 is left unpaired.

So, 243 is not a perfect square.

Question 2.

Show that each of the following numbers is a perfect square. Also, find the number whose square is the given number.

(i) 1296

(ii) 1764

(iii) 3025

(iv) 3969

Solution:

(i) 1296

 $\frac{2|1296}{2|648} \\
 \overline{7|324} \\
 \overline{7|162} \\
 \overline{3|81} \\
 \overline{3|27} \\
 \overline{3|9} \\
 \overline{3|3} \\
 \overline{1}$

 $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

After pairing the same prime factors, we see that no factor is left.

So, 1296 is a perfect square and is the perfect square of $2 \times 2 \times 3 \times 3 = 36$

(ii) 1764

2 1764	
2 882	
7 441	
7 63	
3 9	
3 3	
1	

 $= 2 \times 2 \times 3 \times 3 \times 7 \times 7$

After pairing the same factors, no factor is left.

So, 1764 is a perfect square and

1764 is the perfect square of $2 \times 3 \times 7 = 42$

(iii) 3025

 $\frac{5|3025}{5|605}$ $\frac{5|605}{11|121}$ $\frac{11|11}{1}$

 $= 5 \times 5 \times 11 \times 11$

After pairing the same prime factors, we see that no factor is left.

So, 3025 is a perfect square and is the perfect square of $5 \times 11 = 55$

(iv) 3969

 $\frac{3|3969}{3|1323}$ $\frac{3|441}{3|147}$ $\frac{7|49}{7|7}$ $\frac{7|7}{1}$

 $= 3 \times 3 \times 3 \times 3 \times 7 \times 7$

After pairing the same prime factors, we see that no factor is left.

So, 3969 is a perfect square and is the square of $3 \times 3 \times 7 = 63$

Question 3.

Find the smallest natural number by which 1008 should be multiplied to make it a perfect square.

Solution:

1008

2 1008								
2 504								
2 252								
2 126								
3 63								
3 21								
7 7								
1								
$= 2 \times$	$2 \times$	2>	< 2	×	3 >	x 3	3 ×	7

After pairing the same kind of prime factor, one factor 7 is left.

So, by multiplying 1008 by 7

We shall get a perfect square

Required smallest number = 7

Question 4.

Find the smallest natural number by which 5808 should be divided to make it a perfect square. Also, find the number whose square is the resulting number.

Solution:

5808

2 5808
2 2904
2 1452
2 726
3 363
11 121
11 11
1

 $= 2 \times 2 \times 2 \times 2 \times 3 \times 11 \times 11$

After pairing the same kind of prime factors, we see that factor 3 is left.

So, by dividing the number by 3, we get a perfect square.

The square root of the resulting number

 $= 2 \times 2 \times 11 = 44$

Ex 3.2

Question 1.

Write five numbers which you can decide by looking at their one's digit that they are not square numbers.

Solution:

We know that a number which ends with the digits 2,3, 7 or 8 at its unit places, is not a perfect square.

For example:

372, 563, 1 11,978, 1282 are not square numbers.

Question 2.

What will be the unit digit of the squares of the following numbers?

(i) 951

(ii) 502

(iii) 329

(iv) 643

(v) 5124

(vi) 7625

(vii) 68327

(viii) 95628

(ix) 99880

(x) 12796

Solution:

The unit digit of the square of the following numbers will be

(i) 951: Its square will have unit digit = 1

(ii) 502: Its square will have unit digit = 4

(iii) 329: Its square will have unit digit =

(iv) 643: Its square will have unit digit = 9

(v) 5124: Its square will have unit digit = 6

(vi) 7625: Its square will have unit digit = 5

(vii) 68327: Its square will have unit digit = 9

(viii) 95628: Its square will have unit digit = 4

(ix) 99880: Its square will have unit digit = 0

(x) 12796: Its square will have unit digit = 6

Question 3.

The following numbers are obviously not perfect. Give reason.

(i) 567

(ii) 2453

(iii) 5298

(iv) 46292

(v) 74000

Solution:

We know that if the square of a number does not have 2,3, 7,8 or 0 (in an odd number) as its unit digit.

So, the squares 567, 2453,5208, 46292 and 74000 can't be the perfect squares as they have 7,2, 8,2 digits at the unit place.

Question 4.

The square of which of the following numbers would be an odd number or an even number? Why?

(i) 573

(ii) 4096

(iii) 8267

(iv) 37916

Solution:

We know that the square of an odd number is odd and a square of an even number is even. Therefore:

(i) 573,

(iii) 8262 are odd numbers.

So, their squares will also be an odd number and

(ii) 4096 and

(iv) 37916 are even numbers.

So, their square will be also even-numbered.

Question 5.

How many natural numbers lie between the square of the following numbers?

(i) 12 and 13

(ii) 90 and 91

Solution:

(i) Numbers of natural number between the squares of 12 and 13

 $= (13^2 - 12^2) - 1$ = (13 + 12 - 1)= 25 - 1 = 24

(ii) Between 90 and 91

 $= (91^{2} - 90^{2}) - 1$ = (91 + 90 - 1)= 181 - 1 = 180

Question 6. Without adding, find the sum. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 Solution: Find the sum at (i) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = n^2$ Here n = 8Sum $= (8)^2 = 64$ (ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 = n^2$ Sum = 15^2 (Here n = 15) Sum = 225 Question 7. (i) Express 64 as the sum of 8 odd numbers. (ii) 121 as the sum of 11 odd numbers. Solution: (i) 64 as the sum of odd number $=(8)^2 = n^2$ = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 (Here n = 8) (ii) $121 = (11)^2$ $= 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 = n^{2}$ (Here n = 11) Question 8.

Express the following as the sum of two consecutive integers:

(i) 19²

(ii) 33²

(iii) 47²

Solution:

We know that $n^2 = \frac{n^2 - 1}{2} + \frac{n^2 + 1}{2}$

(When n is odd)

Sum of two consecutive integer

(ii) 19²

$$= \frac{19^2 - 1}{2} + \frac{19^2 + 1}{2} (\because 19^2 = 361)$$
$$= \frac{361 - 1}{2} + \frac{361 + 1}{2} = 180 + 181$$

(ii) 33²

$$= \frac{33^2 - 1}{2} + \frac{33^2 + 1}{2} (\because 33^2 = 1089)$$

$$= \frac{1089 - 1}{2} + \frac{1089 + 1}{2}$$

$$= \frac{1088}{2} + \frac{1090}{2} = 544 + 545$$

(iii) $47^2 = \frac{47^2 - 1}{2} + \frac{47^2 + 1}{2} (\because 47^2 = 2209)$
$$= \frac{2209 - 1}{2} + \frac{2209 + 1}{2}$$

$$= \frac{2208}{2} + \frac{2210}{2} = 1104 + 1105$$

Question 9.

Find the squares of the following numbers without actual multiplication:

(i) 31

(ii) 42

(iii) 86

(iv) 94

Solution:

Using
$$(a+b)^2 - a^2 + 2ab + b^2$$

(i)
$$(31)^2 = (30 + 1)^2$$

= $(30)^2 + 2 \times 30 + 1 + (1)^2$
= $900 + 60 + 1 = 961$
(ii) $(42)^2 = (40 + 2)^2$

$$= (40)^2 + 2 \times 40 \times 2 + (2)^2$$

$$= 1600 + 160 + 4 = 1764$$

(iii)
$$(86)^2 = (80+6)^2$$

.

$$= (80)^2 + 2 \times 80 \times 6 + (6)^2$$

$$= 6400 + 960 + 36 = 7396$$

(iv)
$$(94)^2 = (90+4)^2$$

$$= (90)^2 + 2 \times 90 \times 4 + (4)^2$$

$$= 8100 + 720 + 16 = 8836$$

Question 10.

Find the squares of the following numbers containing 5 in unit's place:

(i) 45

(ii) 305

(iii) 525

Solution:

(i) $(45)^2 = (n5)^2$

= (n) (n + 1) hundred $+ (5)^{2}$

 $= 4 \times 5$ hundred + 25

$$= 2000 + 25 = 2025$$

(ii) $(305)^2 = (30 \times 31)$ hundred + 25

$$= 93000 + 25 = 93025$$

(iii) $(525)^2 = (52 \times 53)$ hundred + 25

$$= 275600 + 25 = 275625$$

Question 11.

Write a Pythagorean triplet whose one number is

(i) 8

(ii) 15

(iii) 63

(iv) 80

Solution:

Pythagoras triplet whose one number is

(i) 8

Let n = 8, then the triplet will be

```
2n, n^2 - 1, n^2 + 1
```

If 2n = 8, then $n = \frac{8}{2} = 4$

 $\therefore n^2 - 1 = 4^2 - 1 = 16 - 1 = 15$

and $n^2 + 1 = 4^2 + 1 = 16 + 1 = 17$

Triplet is 8.15,17

(ii) 15

Let 2n = 15, then $n = \frac{n}{2}$ which is not possible Or $n^2 - 1 = 15 \Rightarrow n^2 = 15 + 1 = 16 = (4)^2$ Now, $2n = 2 \times 4 = 8$ $n^2 - 1 = 15$ $n^2 + 1 = 4^2 + 1 = 16 + 1 = 17$ Triplet is 8,15, 17 (iii) 63 Let $n^2 - 1 = 63 \Rightarrow n^2 = 63 + 1 = 64 = (8)^2$

```
\therefore n = 8
```

Now, $2n = 2 \times 8 = 16$ $n^2 - 1 = 63$ $n^2 + 1 = 8^2 + 1 = 64 + 1 = 65$ Triplet is 16,63, 65 (iv) 80 Let $2n = 80 \Rightarrow n = \frac{80}{2} = 40$ $\therefore n^2 - 1 = 40^2 - 1 = 1600 - 1 = 1599$ and $n^2 + 1 = 402 + 1 = 1600 + 1 = 1601$ \therefore Triplet is 80, 1599, 1601

Question 12.

Observe the following pattern and find the missing digits:

```
21^{2} = 441

201^{2} = 40401

2001^{2} = 4004001

20001^{2} = 4...4...1

200001^{2} = ...
```

Solution:

 $21^2 = 441$

 $201^2 = 40401$

```
2001^2 = 4004001
```

```
Similarly, 20001^2 = 400040001
```

```
200001^2 = 40000400001
```

Question 13.

Observe the following pattern and find the missing digits:

 $9^{2} = 81$ $99^{2} = 9801$ $999^{2} = 998001$ $9999^{2} = 99980001$ $99999^{2} = 9...8...01$ $999999^{2} = 9...0...1$

Solution:

```
9^{2} = 81

99^{2} = 9801

999^{2} = 998001

9999^{2} = 99980001

99999^{2} = 9999800001

999999^{2} = 999998000001
```

Question 14.

Observe the following pattern and find the missing digits:

 $7^{2} = 49$ $67^{2} = 4489$ $667^{2} = 444889$ $6667^{2} = 44448889$ $66667^{2} = 4...8...9$ $666667^{2} = 4...8...8...$

Solution:

 $7^2 = 49$

 $67^2 = 4489$

 $667^2 = 444889$

 $6667^2 = 44448889$

Similarly, $66667^2 = 4444488889$

```
666667^2 = 444444888889
```

Ex 3.3

Question 1.

By repeated subtraction of odd numbers starting from 1, find whether the following numbers are perfect squares or not? If the number is a perfect square then find its square root:

(i) 121

(ii) 55

(iii) 36

(iv) 90

Solution:

(i) Square root of 121

- 121 1 = 120 120 3 = 117 117 5 = 112 112 7 = 105 105 9 = 96 96 11 = 85 85 13 = 72 72 15 = 57 57 17 = 40 40 19 = 21 21 21 = 0Square root of 121 is 11
- 121 is a perfect square.

(ii) Square root of 55

- 55 1 = 5454 - 3 = 51
- 51 5 = 46
- 46 7 = 39
- 39 9 = 30
- 30 11 = 19
- 19 13 = 6
- 6 15 = not possible
 - 55 is not a perfect square.

(iii) Square root of 36

36 - 1 = 3535 - 3 = 3232 - 5 = 2727 - 7 = 2020 - 9 = 1111 - 11 = 0

36 is a perfect square and its square root is 6.

(iv) Square root of 90

- 90 1 = 8989 3 = 8686 5 = 8181 7 = 7474 9 = 6565 11 = 5454 13 = 41
- 41 15 = 26
- 26 17 = 9
- 9 19 = not possible.
 - 90 is not a perfect square.
- (iv) Square root of 90

90 - 1 = 89

- 89 3 = 8686 5 = 8181 7 = 7474 9 = 6565 11 = 5454 13 = 4141 15 = 26
- 26 17 = 9
- 9 19 = not possible.
 - 90 is not a perfect square.

Question 2.

Find the square roots of the following numbers by prime factorization method:

(i) 784

(ii) 441

(iii) 1849

(iv) 4356

- (v) 6241
- (vi) 8836
- (vii) 8281

(viii) 9025

Solution:

(i) Square root of 784

 $\frac{\frac{2|784}{2|392}}{\frac{2|196}{\frac{2|98}{\frac{7|49}{\frac{7|7}{1}}}}}$ $\sqrt{784} = \sqrt{2 \times 2 \times 2 \times 2 \times 7 \times 7}$

 $= 2 \times 2 \times 7 = 28$

(ii) Square root of 441

3 441	
7 147	
7 49	
7 7	
1	

$$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$$

$$= 3 \times 7 = 21$$

(iii) Square root of 1849

$$\frac{\frac{43|1849}{43|43}}{\sqrt{1849}} = \sqrt{43 \times 43} = 43$$

(iv) Square root of 4356

$$\frac{2|4356}{2|2178}$$

$$\frac{3|1089}{3|363}$$

$$\frac{11|121}{11}$$

$$\frac{11|11}{1}$$

$$\sqrt{4356} = \sqrt{2 \times 2 \times 3 \times 3 \times 11 \times 11}$$

$$= 2 \times 3 \times 11 = 66$$

(v) Square root of 6241

$$\frac{79|6241}{\frac{79|79}{1}}$$

$$\sqrt{6241} = \sqrt{79 \times 79}$$
= 79

(vi) Square root of 8836

$$\frac{2|8836}{2|4418}$$

$$\frac{47|2209}{47|47}$$

$$\frac{47|47}{1}$$

$$\sqrt{8836} = \sqrt{2 \times 2 \times 47 \times 47}$$

$$= 2 \times 47 = 94$$
(vii) Square root of 8281

7 8281				
7 1183				
13 169				
13 13				
1				
$\sqrt{8281}$	$=\sqrt{7}$	$\times 7 \times$	13 ×	13

 $= 7 \times 13 = 91$

(viii) Square root of 9025

5 9025
5 1805
19 361
19 19
1
$\sqrt{9025} = \sqrt{5 \times 5 \times 19 \times 19}$
$= 5 \times 19 = 95$

Question 11.

A gardener has 1400 plants. He wants to plant these in such a way that the number of rows and number of columns remains the same. Find the minimum number of plants he needs more for this.

Solution:

Total plants = 1400

Number of columns = Number of rows

Now, taking the square root of 1400, we see that $(37)^2 < 1400$

```
We shall take (38)^2 = 1444
```

So, we need 1444 - 1400 = 44 plants more

Hence required plants = 44

Question 12.

There are 1000 children in a school. For a P. T. drill they have to stand in such a way that the number of rows is equal to a number of columns. How many children would be left out in this arrangement?

Solution:

Number of total children in a school = 1000

For PT. drill, the children have to stand in such a way that.

Number of rows = Number of columns

Now take the square root of 1000, we see that (39 left as remainder).

Left out children = 39

Question 13.

Amit walks 16 m south from his house and turns east to walk 63 m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?

Solution:

Amit walks 16 m south from his house at then turn to east 63 m.

O is the house and A and B are the places.

OA = 16 m, AO = 63 m



$$OB^2 = OA^2 + AB^2$$

 $= 16^2 + 63^2$ (Pythagoras Theorem)

= 256 + 3969 = 4225

$$\therefore OB = \sqrt{4225} = 65$$

$$\frac{65}{6)4225}$$

$$\frac{36}{125)625}$$

$$\frac{625}{0}$$

He is 65 m from his house.

So, he has to walk 65 m to reach his house.

Question 14.

A ladder 6 m long leaned against a wall. The ladder reaches the wall to a height of 4.8 m. Find the distance between the wall and the foot of the ladder.

Solution:

Length of a ladder = 6 m

It reaches the wall at a height of 4.8 m

AB is the ladder and AC is the height of the wall

AB = 6 m, AC = 4.8 m



Now the distance between the foot of the ladder and the wall is BC Now,

 $AB^{2} = AC^{2} + BC^{2} \text{ (Pythagoras Theorem)}$ $6^{2} = 4.8^{2} + BC^{2}$ $BC^{2} = 6^{2} - 4.8^{2} = 36 - 23.04 = 12.96$ $\therefore BC = \sqrt{12.96} = 3.6m$ $\frac{3.6}{3)12.96}$ $\frac{9}{66)396}$ $\frac{396}{0}$

Question 4.

For each of the following numbers, find the smallest natural number by which it should be multiplied so as to get a perfect square. Also, find the square root of the square number so obtained:

(i) 588

(ii) 720

(iii) 2178

(iv) 3042

(v) 6300

Solution:

```
(i) 588 = 2 \times 2 \times 3 \times 7 \times 7
```

 $\begin{array}{r}
 2|588 \\
 2|94 \\
 3|147 \\
 \overline{7|49} \\
 \overline{7|7} \\
 1
 \end{array}$

Pairing the same kind of factors, we see that one factor 3 is left unpaired.

In order to make it a pair. it must be multiplied by 3.

Required least number = 3

and the square root of $588 \times 3 = 1764$

 $= 2 \times 3 \times 7 = 42$

(ii) $720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

 $\frac{2|720}{2|360} \\
 \frac{2|360}{2|180} \\
 \frac{2|90}{3|45} \\
 \frac{3|15}{5|5} \\
 1$

Pairing the same kind of factors, we see that one factor 5 is left unpaired.

In order to make it a pair, it must be multiplied by 5.

Required least number = 5

and square root of $720 \times 5 = 3600$

 $= 2 \times 2 \times 3 \times 5 = 60$

(ii) $2178 = 2 \times 3 \times 3 \times 11 \times 11$

 $\frac{2|2178}{3|1089} \\
 \frac{3|363}{11|121} \\
 \frac{11|11}{1} \\
 1$

Pairing the same kind of factors, one factor

2 is left unpaired.

The required least number to be multiplied = 2

and the square root of $2178 \times 2 = 4356$

 $= 2 \times 3 \times 11 = 66$

(iv) $3042 = 2 \times 3 \times 3 \times 13 \times 13$

 $\begin{array}{r}
 2|3042 \\
 3|1521 \\
 \hline
 3|507 \\
 13|169 \\
 \hline
 13|13 \\
 1
 \end{array}$

Pairing the same kind of factors one factor 2 is left unpaired.

The required least number to be multiplied = 2.

The square root of $3042 \times 2 = 6084$

 $= 2 \times 3 \times 13 = 78$

(v) $6300 = 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7$

2 6300
2 3150
3 1575
3 525
3 175
5 35
7 7
1

Pairing the same kind of factors one factor 7 is left unpaired

The required least number to be multiplied = 7

The square root of $6300 \times 7 = 44100$

 $= 2 \times 3 \times 5 \times 7 = 210$

Question 5.

For each of the following numbers, find the smallest natural number by which it should be divided so that this quotient is a perfect square. Also, find the square root of the square number so obtained:

(i) 1872

(ii) 2592

- (iii) 3380
- (iv) 16224
- (v) 61347

Solution:

(i) $1872 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 13$

 $\begin{array}{r}
 2|1872 \\
 \hline
 2|936 \\
 2|468 \\
 2|234 \\
 \hline
 3|117 \\
 \hline
 3|39 \\
 13|13 \\
 1
 \end{array}$

Pairing the same kind of factors, one factor 13 is left unpaired.

Required least number = 13.

So, 1872 be divided by 13, the resultant number will be a perfect square.

Resultant number $= 1872 \div 13 = 144$

and square root $= 2 \times 2 \times 3 = 12$

(ii) $2592 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

2 2592
2 1296
2 648
2 324
2 162
3 81
3 27
3 9
3 3
1

Pairing the same kind of factors, one factor 2 is left unpaired.

Required least number = 2

and the resultant number $= 2592 \div 2 = 1296$

and square root $= 2 \times 2 \times 3 \times 3 = 36$

(iii) $3380 = 2 \times 2 \times 5 \times 13 \times 13$

 $\frac{2|3380}{2|1690} \\
 \frac{5|845}{13|169} \\
 \frac{13|13}{1}$

Pairing the same kind of factors, one factor 5 is left unpaired.

The required least number = 5

and the resultant number $= 3380 \div 5 = 676$

and the square root $= 2 \times 13 = 26$

(iv) $16224 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 13 \times 13$

2 16224
2 8112
2 4056
2 2028
2 1014
3 507
13 169
13 13
1

Pairing the same kind of factors, two factors 2 and 3 are left unpaired.

So, the required least number $= 2 \times 3 = 6$

and the resultant number = 16224 - 6 = 2704

and square root $= 2 \times 2 \times 13 = 52$

(v) $61347 = 3 \times 11 \times 11 \times 13 \times 13$

 $\frac{3|61347}{11|20449} \\
 \underline{11|1859} \\
 \underline{13|169} \\
 \underline{13|13} \\
 1$

Pairing the same kind of factors, one factor 3 is left unpaired.

So, the required least number is 3.

The resultant number $= 61347 \div 3 = 20449$

And the square root $= 11 \times 13 = 143$

Question 6.

Find the smallest square number that is divisible by each of the following numbers:

(i) 3, 6,10, 15

(ii) 6, 9,27, 36

(iii) 4,7, 8,16

Solution:

(i) 3, 6,10, 15

The number which is divisible by

3, 6,10, 15 = The LCM of 3, 6,10, 15

 $\frac{2|3, 6, 10, 15}{3|3, 3, 5, 15}$ $\frac{5|1, 1, 5, 5}{1, 1, 1, 1}$ $= 2 \times 3 \times 5 = 30$

and smallest square number which is divisible by $30 = 30 \times 30 = 900$

(ii) 6, 9,27, 36

The number which is divisible by 6, 9,27, 36 is their LCM

 $\frac{3|6,9,27,36}{3|2,3,9,12}$ $\frac{2|2,1,3,4}{1,1,3,2}$ = 3 × 3 × 2 × 2 × 3 = 108

and the smallest square

 $= 108 \times 3 = 324$

(iii) 4,7, 8,16

The number which is divisible by 4, 7,8, 16 = LCM of their numbers

 $\frac{2|4,7,8,16}{2|2,7,4,8}$ $\frac{2|1,7,2,4}{1,7,1,2}$ $= 2 \times 2 \times 2 \times 2 \times 7 = 112$

The smallest square $= 112 \times 7 = 784$

Question 7.

4225 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Solution:

Total number of plants = 4225

The number of rows = Number of the plant in each row.

Number of rows = Square root of 4225

 $\frac{5|4225}{5|845} \\ \frac{13|169}{13|13} \\ = \sqrt{5 \times 5 \times 13 \times 13} \\ = 5 \times 13 = 65$

Number of rows = 65

and number of plants in each row = 65

Question 8.

The area of a rectangle is 1936 sq. m. If the length of the rectangle is 4 times its breadth, find the dimensions of the rectangle.

Solution:

Area of a rectangle = 1936 sq. m

Let breadth = x m

Then length = 4x m

$$\therefore 4x^{2} = 1936 \Rightarrow 4 \times 2 = 1936$$
$$x^{2} = \frac{1936}{4} = 484$$
$$x = \sqrt{484}$$
$$= \sqrt{2 \times 2 \times 11 \times 11}$$

2 484
2 242
11 121
11 11
1

Length $= 4x = 4 \times 22 = 88m$

and breadth = x = 22 m

Question 9.

In a school a P. T. teacher wants to arrange 2000 students in the form of rows and columns for P. T. display. If the number of rows is equal to number of columns and 64 students could not be accommodated in this arrangement. Find the number of rows.

Solution:

Total number of students in a school = 2000

The P. T. teacher wants to arrange them in such a way.

The numbers of rows = Number of students in each row,

By doing this, 64 students are lefts

Required number of students = 2000 - 64 = 1936

```
and number of rows = \sqrt{1936}

\frac{2|1936}{2|968}
\frac{2|484}{2|242}
\frac{11|121}{11|11}
= \sqrt{2 \times 2 \times 2 \times 2 \times 11 \times 11}
= 2 \times 2 \times 11 = 44
```

Question 10.

In a school, the students of class VIII collected ₹ 2304 for a picnic. Each student contributed as many rupees as the number of students in the class. Find the number of students in the class.

Solution:

In a school; students of class VIII collected for contribution = 2304

Number of students = Number of rupees contribute each student

Number of students $= \sqrt{2304}$
2 2304
2 1152
2 576
2 288
2 144
2 72
2 36
2 18
3 9
3 3
1
$= \sqrt{2 \times 2 \times 3 \times 3}$
$= 2 \times 2 \times 2 \times 2 \times 3 = 48$

Hence number of students in class VIII = 4811.

Question 11.

The product of two numbers is 7260. If one number is 15 times the other number, find the numbers.

Solution:

Product of two numbers = 7260

Let one number = x

Then second = 15x

$$\therefore 15x \times x = 7260 \Rightarrow 15x^2 = 7260$$

$$x^{2} = \frac{7260}{15} = 484$$

$$x = \sqrt{484}$$

$$= 2 \times 11 = 22$$

$$= \sqrt{2 \times 2 \times 11 \times 11}$$

$$\frac{2|484}{2|242}$$

$$\frac{11|121}{11|11}$$

One number = 22

and second number $= 22 \times 15 = 330$

Question 12.

Find three positive numbers in the ratio 2: 3: 5, the sum of whose squares is 950.

Solution:

Ratio in three numbers = 2: 3: 5

Sum of their square = 950

Let first number = 2x

Second number = 3x

and third number = 5x

$$\therefore (2x)^{2} + (3x)^{2} + (5x)^{2} = 950$$

$$\Rightarrow 4x^{2} + 9x^{2} + 25x^{2} = 950$$

$$\Rightarrow 38x^{2} = 950 \Rightarrow x^{2} = \frac{950}{38} = 25$$

$$x = \sqrt{25} = 5$$

First number $= 2 \times 5 = 10$

Second number $= 3 \times 5 = 15$

Third number $= 5 \times 5 = 25$

Question 13.

The perimeter of two squares is 60 metres and 144 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

Solution:

Perimeter of first square = 60 m

Side
$$=\frac{60}{4} = 15m$$

and second square = 144

and side
$$=\frac{144}{4}=36m$$

Sum of perimeters of two squares

$$= 60 + 144 = 204m$$

and sum of areas of these two squares

$$= 152m^{2} + 362m^{2}$$
$$= 225m^{2} + 1296m^{2} = 1521m^{2}$$
Area of third square = $1521m^{2}$

Side =
$$\sqrt{Area} = \sqrt{1521}m$$

= $\sqrt{3 \times 3 \times 13 \times 13}$
 $\frac{3|1521}{3|507}$
 $\frac{13|169}{13|13}$
1

$$= 3 \times 13 = 39m$$

Perimeter
$$= 3 \times Side$$

 $= 4 \times 39 = 156m$

Ex 3.4

Question 1.

Find the square root of each of the following by division method:

(i) 2401

(ii) 4489

(iii) 106929

- (iv) 167281
- (v) 53824

(vi) 213444

Solution:

(i)
$$\sqrt{2401} = 49$$

 49
 $4)2401$
 16
 $89)801$
 $\frac{801}{0}$
(ii) $\sqrt{4489} = 67$
 67
 $6)4489$
 36
 $127)889$
 $\frac{889}{0}$
(iii) $\sqrt{106929} = 327$

207
321
2)106929
9
62)169
124
647)4529
4529
0
0
(iv) $\sqrt{167281} = 409$
409
2)167281
16
809)7281
7281
0
(v) $\sqrt{53824} = 232$
232
2,15382
4
43)138
129
462)02
402)92
92
0
(vi) $\sqrt{213444} = 462$
462
4)213444
16
96)524
0UJJJ4 516
010
922)1844
1844
0
Question 2.

Find the number of digits in the square root of each of the following (without any calculation) :

(i) 81

(ii) 169

(iii) 4761

(iv) 27889

(v) 525625

Solution:

(i) 81

In 81, a group of two's is 1.

Its square root has one digit.

(ii) 169

In 169, groups of two's are 2. Its square root has two digits. 4761

In 4761, groups of two's are 2. Its square root has two digits.

(iii) 27889

In 27889, groups of two's are 3.

Its square root has 3 digits.

(iv) 525625

In 525625, groups of two's are 3.

Its square root has 3 digits.

Question 3.

Find the square root of the following decimal numbers by division method:

(i) 51.84

(ii) 42.25

(iii) 18.4041

(iv) 5.774409

Solution:

(i) $\sqrt{51.84} = 7.2$ 7.2 7)51.84 49 142)284 284 0 (ii) $\sqrt{42.25} = 6.5$ 6.5 4)42.25 36 125)625 625 0 (iii) $\sqrt{18.4041} = 4.29$ 4.29 4)18.4041 16 82)240 164 849)7641 7641 0 (iv) $\sqrt{5.774409} = 2.403$ $2.403 \\ 4 \\ 2)5.774436 \\ 4 \\ 44)177 \\ 176 \\ 4803)14409 \\ 14409 \\ 0 \\ 0$

Question 4.

Find the square root of the following numbers correct to two decimal places:

(i) 645.8

(ii) 107.45

(iii) 5.462

(iv) 2

(v) 3

Solution:

(i) $\sqrt{645.8} = 25.41$

```
25.41
      2)645.8000
          4
        45)245
         225
      504)2080
         2016
      5081) 6400
         5081
         1319
(ii) \sqrt{107.4} = 10.36
         10.36
      1)107.4000
           1
       203)0740
         609
      2066)13100
         12396
         704
```

(iii) $\sqrt{5.462} = 2.337 = 2.34$

2.337
2)5.462000
4
463)1720
1389
4667)33100
32669
431
(iv) $\sqrt{2} = 1.41$
1.41
1)2.0000
1
24)100
96
281)400
281
119
(v) $\sqrt{3} = 1.73$
1.73
1)3.0000
1
27) 200
189
343)1100
1029
71

Question 3.

Find the square roots of the following numbers by prime factorization method:

(i) $9\frac{67}{121}$ (ii) $17\frac{13}{36}$ (iii) 1.96 (iv) 0.0064 Solution: (i) $9\frac{67}{121}$ $=\frac{9 \times 121 + 67}{121}$ $=\frac{1089+67}{121}$ $=\frac{1156}{121}$ $\therefore \sqrt{\frac{1156}{121}}$ $=\frac{\sqrt{1156}}{\sqrt{121}}$ 2|1156 2|578 17|289 $\frac{17|17}{1}$ $=\frac{\sqrt{2\times2\times17\times17}}{\sqrt{11\times11}}$

$$= \frac{\sqrt{11 \times 10^{-1}}}{\sqrt{11 \times 10^{-1}}}$$
$$= \frac{34}{11}$$
$$= 3\frac{1}{11}$$

(ii)
$$17\frac{13}{36}$$

$$=\frac{17 \times 36 + 13}{36}$$

$$=\frac{612 + 13}{36}$$

$$=\frac{625}{36}$$

$$\therefore \sqrt{\frac{625}{36}}$$

$$=\frac{\sqrt{625}}{\sqrt{36}}$$

$$\frac{5|625}{5|125}$$

$$\frac{5|25}{5|25}$$

$$\frac{5|25}{5|5}$$

$$\frac{1}{1}$$

$$\frac{2|36}{2|18}$$

$$\frac{3|9}{3|3}$$

$$1$$

$$=\frac{\sqrt{5 \times 5 \times 5 \times 5}}{\sqrt{2 \times 2 \times 3 \times 3}}$$

$$=\frac{5 \times 5}{2 \times 3}$$

$$=\frac{25}{6}$$

$$=4\frac{1}{6}$$
(iii) $1.96 = \frac{196}{100}$

$$\therefore \sqrt{\frac{196}{100}}$$

$$=\frac{\sqrt{196}}{\sqrt{100}}$$

$$\frac{2|196}{2|98}$$

$$\frac{7|49}{7|49}$$

$$\frac{7|7}{1}$$

	2 100
	2 50
	5 25
	5 5
	1
	$=\frac{\sqrt{2\times2\times7\times7}}{\sqrt{2\times2\times5\times5}}$
	$=\frac{2\times7}{2\times5}$
	$=\frac{14}{10}$
	= 1.4
(iv)	$0.0064 = \frac{64}{10000}$
	$=\sqrt{\frac{64}{10000}}$
	$=\frac{\sqrt{64}}{\sqrt{10000}}$
	2 10000
	2 5000
	2 2500
	2 1250
	5 625
	5 125
	5 25
	5 5
	1
	$=\frac{\sqrt{2\times2\times2\times2\times2\times2}}{\sqrt{2\times2\times2\times2\times2\times2}}$
	$\sqrt{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5}$
	$=\frac{2\times2\times2}{2\times2\times5\times5}$
	$=\frac{8}{100}$
	= 0.08

Question 5.

Find the square root of the following fractions by division method:

- (i) $\frac{841}{1521}$
- (ii) $8\frac{257}{529}$ (iii) $16\frac{169}{441}$

Solution:

(i)
$$\sqrt{\frac{841}{1521}}$$

$$= \frac{\sqrt{841}}{\sqrt{1521}}$$

$$= \frac{29}{39}$$

$$\frac{29}{2)841}$$

$$\frac{4}{49)441}$$

$$\frac{441}{0}$$

$$\frac{39}{3)1521}$$

$$9$$

$$69)\overline{621}$$

$$\frac{621}{0}$$
(ii) $\sqrt{8\frac{257}{529}}$

$$= \sqrt{\frac{4232 + 257}{529}}$$

$$= \sqrt{\frac{4489}{529}}$$

$$= \frac{\sqrt{4489}}{\sqrt{529}}$$

	$=\frac{67}{23}$	
	$=2\frac{21}{23}$	
	23	
	2)529 4	
	43)129	
	129	
	0	
	67	
	6)4489	
	36	
	889	
	0	
(iii)	$\sqrt{16\frac{169}{441}}$	
	$=\sqrt{\frac{7056+169}{441}}$	
	$=\sqrt{\frac{7225}{441}} = \frac{\sqrt{722}}{\sqrt{441}}$	5
	21	
	2)441 4	
	41)41	
	41	
	0	
	85	
	8)7225	
	165)825	
	825	
	0	

Question 6.

Find the least number which must be subtracted from each of the following numbers to make them a perfect square. Also find the square root of the perfect square number so obtained:

(i) 2000 (ii) 984 (iii) 8934 (iv) 11021 Solution: (i) 2000 444)20001684)40033664

Taking square root, we see that 64 is left as remainder

So, subtracting 64 from 2000

We get 1936, which is a perfect square and its square root is 44.

(ii) 984

31 $3\overline{)984}$ 9 $61\overline{)84}$ $\underline{61}$ 23

Taking square root, we see that 23 is left as remainder

So, subtracting 23 from 984

We get 984 - 23 = 961 which is a perfect square and its square root is 31.

(iii) 8934

94
9)8934
81
834
736
98

Taking square root, we see that 98 is left as remainder

So, subtracting 98 from 894

We get 8934 - 98 = 8836 which is a perfect square and its square root is 94.

(iv) 11021

 $\begin{array}{r}
 104 \\
 1)\overline{11021} \\
 1 \\
 204)\overline{1021} \\
 \underline{816} \\
 205 \\
 \end{array}$

Taking square root, we see that 205 is left as remainder

So, subtracting 205 from 11021

We get 11021 - 205 = 10816

Which is a perfect square and its square root is 104.

Question 7.

Find the least number which must be added to each of the following numbers to make them a perfect square. Also find the square root of the perfect square number so obtained:

(i) 1750

(ii) 6412

- (iii) 6598
- (iv) 8000

Solution:

(i) 1750

 $\begin{array}{r}
 42 \\
 4\overline{)1750} \\
 16 \\
 82\overline{)150} \\
 -164 \\
 -14 \\
\end{array}$

Taking square root, we see that

 $(41)^2$ is less than 1750

So, we take $(42)^2$, we get

 $164 - 150 = 14 \, less$

So, by adding 14, we get a square of 42 which is 1764.

(ii) 6412

 $81 \\ 8)6412 \\ 64 \\ 161)12 \\ -161 \\ -149$

Taking a square of 6412, we see that $(80)^2$ is less than 6412.

So, we shall take $(81)^2$ and we get

161 - 12 = 14 less

So, by adding 149, we get a square of 81 which is 6561

(iii) 6598

 $82 \\ 8)6598 \\ 64 \\ 162)198 \\ -324 \\ 126$

Taking the square root of 6598, we see that $(81)^2$ is less than 6598

So, we shall take $(82)^2$ and we get

324 - 198 = 126 less

By adding 126. we get a perfect square 6724

which is square of 82.

(iv) 8000

 $\begin{array}{r}
 89 \\
 8)8000 \\
 64 \\
 169)1600 \\
 \underline{1521} \\
 79 \\
 79$

Taking square root of 8000, we see that $(89)^2$ is less than 8000.

We shall take $(90)^2 = 8100$

which 8100 - 8000 = 100 is less

So, by adding 100 we get $(90)^2 = 8100$ which is a perfect square.

90)8000(90)-8100 100

Question 8.

Find the smallest four-digit number which is a perfect square.

Solution:

Smallest 4-digit number = 1000

 $\begin{array}{r}
 31 \\
 3)1000 \\
 9 \\
 61)100 \\
 61 \\
 39
 \end{array}$

Taking square root, we see that 39 is left.

If we subtract any number from 1000, we get a 3-digit number

So, we shall take $(32)^2 = 1024$

So, 1024 - 1000 = 24 is to be added to get a perfect square of a least 4-digit number

```
Hence required 4-digit smallest number = 1024
```

Question 9.

Find the greatest number of six digits which is a perfect square.

Solution:

Greatest 6-digit number = 999999

```
999

9) 9999999

81

189) 1899

1701

1989) 19899

17901

1998
```

Taking square root of 999999, we see that 1998 is left

Subtracting 1998 from 999999 we get 998001 which is a perfect square.

Hence, required 6-digit greatest number = 998001

Question 10.

In a right triangle ABC, $\angle B = 90^{\circ}$.

- (i) If AB = 14 cm, BC = 48 cm, find AC.
- (ii) If AC = 37 cm, BC = 35 cm, find AB.

Solution:



(i) In right angled triangle ABC

$$AB = 14 \text{ cm} \text{ and } BC = 48 \text{ cm}$$

$$AC^2 = AB^2 + BC^2$$
 (Pythagoras Theorem)

$$= 14^2 + 48^2$$

 $\frac{25}{\overline{0}}$

= 196 + 2304 = 2500

$$\therefore AC = \sqrt{2500} = 50 \ cm$$

$$\frac{50}{5)2500}$$

(ii) In $\triangle ABC, B = 90^{\circ}$

AC = 37 cm, BC = 35 cm



 $AC^{2} = AB^{2} + BC^{2} \text{ (Pythagoras Theorem)}$ $37^{2} = AB^{2} + 35^{2}$ $1369 = AB^{2} - 1225$ $AB^{2} = 1369 - 1225 = 144$ $AB = \sqrt{144} = 12cm$ $\frac{12}{1)144}$ $\frac{1}{22)44}$ $\frac{44}{0}$