CHAPTER – 4

CUBES AND CUBE ROOTS

Exercise 4.1

1. Which of the following numbers are not perfect cubes? Give reasons in support of your answer:

(i) 648
(ii) 729
(iii) 8640
(iv) 8000
Solution:

(i) We have,

$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, one factor 3 is left without grouping.

$648 = (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 3$

Thus, 648 is not a perfect cube.

(ii) We have,

$729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3	729
3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left.

$729 = (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

Thus, 729 is a perfect cube.

(iii) We have,

$8640 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

After grouping the prime factors in triplets, it's seen that one factor 5 is left without grouping.

$8640 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5$

Thus, 8640 is not a perfect cube.

(iv) We have,

$8000 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

After grouping the prime factors in triplets, it's seen that no factor is left.

$8000 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (5 \times 5 \times 5)$

Thus, 8000 is a perfect cube.

2. Show that each of the following numbers is a perfect cube. Also, find the number whose cube is the given number:

(i) 1728
(ii) 5832
(iii) 13824
(iv) 35937
Solution:

(i) We have,

$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

$1728 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

Thus, 1728 is a perfect cube and its cube root is $2 \times 2 \times 3 = 12$.

(ii) We have,

$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

5832 = (2 × 2 × 2) × (3 × 3 × 3) × (3 × 3 × 3)

Thus, 5832 is a perfect cube and its cube root is 2 × 3 × 3 = 18

(iii) We have,

13824 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, its seen that no factor is left without grouping.

13824 = (2 × 2 × 2) × (2 × 2 × 2) × (2 × 2 × 2) × (3 × 3 × 3)

Thus, 13824 is a perfect cube and its cube root is 2 × 2 × 2 × 3 = 24.

(iv) We have,

35937 = 3 × 3 × 3 × 11 × 11 × 11

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

After grouping the prime factors in triplets, it's seen that no factor is left without grouping.

$35937 = (3 \times 3 \times 3) \times (11 \times 11 \times 11)$

Thus, 35937 is a perfect cube and its cube root is $3 \times 11 = 33$

3. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube:

(i) 243
(ii) 3072
(iii) 11979
(iv) 19652
Solution:

(i) We have,

$243 = 3 \times 3 \times 3 \times 3 \times 3$

3	243
3	81
3	27
3	9
3	3
	1

After grouping the prime factors in triplets, it's seen that factors 3×3 are left.

$243 = (3 \times 3 \times 3) \times 3 \times 3$

So, in order to complete in a group of 3's, one more factor of 3 is needed. Thus, the smallest number which should be multiplied to 243 in order to make it a perfect cube is 3.

(ii) We have,

$3072 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

2	3072
2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

After grouping the prime factors in triplets, it's seen that factor 2×3 are left ungrouped.

$3072 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2 \times 3$

So, in order to complete them in a group of 3's we need the factors $2 \times 2 \times 3 \times 3$ to be multiplied.

i.e. the factor needed is $2 \times 2 \times 3 \times 3 = 36$

Thus, the smallest number which should be multiplied to 3072 in order to make it a perfect cube is 36.

(iii) We have,

$11979 = 3 \times 3 \times 11 \times 11 \times 11$

3	11979
3	3993
11	1331
11	121
11	11
	1

After grouping the prime factors in triplets, it's seen that factors 3×3 are left without grouping in 3's.

$11979 = 3 \times 3 \times (11 \times 11 \times 11)$

So, in order to complete in a group of 3's, one more factor of 3 is needed.

Thus, the smallest number which should be multiplied to 11979 in order to make it a perfect cube is 3.

(iv) We have,

$19652 = 2 \times 2 \times 17 \times 17 \times 17$

2	19652
2	9826
17	4913
17	289
17	17
	1

After grouping the prime factors in triplets, it's seen that factors 2 × 2 are left ungrouped in 3's.

$19652 = 2 \times 2 \times (17 \times 17 \times 17)$

So, in order to complete it in a triplet one more 2 is needed.

Thus, the smallest number which should be multiplied to 19652 in order to make it a perfect cube is 2.

4. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube:

(i) 1536
(ii) 10985
(iii) 28672
(iv) 13718
Solution:

(i) We have,

$1536 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

After grouping the prime factors in triplets, it's seen that one factor 3 is left without grouping.

$1536 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 3$

So, in order to make it a perfect cube, it must be divided by 3.

Thus, the smallest number by which 1536 must be divided to obtain a perfect cube is 3.

(ii) We have,

$10985 = 5 \times 13 \times 13 \times 13$

5	10985
13	2197
13	169
13	13
	1

After grouping the prime factors in triplet, it's seen that one factor 5 is left without grouping.

$10985 = 5 \times (13 \times 13 \times 13)$

So, it must be divided by 5 in order to get a perfect cube.

Thus, the required smallest number is 5.

(iii) We have,

$28672 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7$

2	28672
2	14336
2	7168
2	3584
2	1792
2	896
2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

After grouping the prime factors in triplets, it's seen that one factor 7 is left without grouping.

$28672 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 7$

So, it must be divided by 7 in order to get a perfect cube. Thus, the required smallest number is 7.

(iv) $13718 = 2 \times 19 \times 19 \times 19$

2	13718
19	6859
19	361
19	19
	1

After grouping the prime factors in triplets, it's seen that one factor 2 is left without grouping.

$13718 = 2 \times (19 \times 19 \times 19)$

So, it must be divided by 2 in order to get a perfect cube.

Thus, the required smallest number is 2.

5. Rahul makes a cuboid of plasticine of sides 3 cm × 3 cm × 5 cm. How many such cuboids will he need to form a cube?

Solution:

Given,

Cuboid with dimensions 3 cm × 3 cm × 5 cm

To form into a cube, number of such cuboid required are

$= \left(\frac{15}{3}\right) \times \left(\frac{15}{3}\right) \times \left(\frac{15}{5}\right)$

$= 5 \times 5 \times 3$

$= 75$

Thus, the number of cuboids required to make a cube is 75.

6. Find the volume of a cubical box whose surface area is 486 cm².

Solution:

Given,

Surface area of a cubical box = 486 cm^2

We know that,

Surface area of a cubical box = $6 \times (\text{side})^2$

So,

Side = $\sqrt{\frac{486}{6}} = 9$ cm

Now, volume = $(\text{Side})^3$

$= (9)^3$

$= 9 \times 9 \times 9 = 729 \text{ cm}^3$

Thus, the volume of the cubical box is 729 cm^3.

7. Which of the following are cubes of even natural numbers or odd natural numbers:

(i) 125
(ii) 512
(iii) 1000
(iv) 2197
(v) 4096
(vi) 6859
Solution:
We know that,

The cube of an even number is even and the cube of an odd number is odd.
Hence,

125, 2197, 6859 are cubes of an odd number and 512, 1000, 4096 are cubes of an even number.

8. Write the ones digit of the cube of each of the following numbers:

(i) 231
(ii) 358
(iii) 419
(iv) 725
(v) 854
(vi) 987
(vii) 752

(viii) 893
Solution:
We know that,

The cube of number having 1, 4, 5, 6 or 9 in unit place will end in 1, 4, 5, 6 or 9

And, if the numbers have:

2 in unit place, then it's cube ends in 8

8 in unit place, then it's cube ends in 2

3 in unit place then it's cube ends in 7

7 in unit place then it's cube ends in 3

0 in unit place then it's cube ends in 0.

So now,

(i) Unit digit of number 231 is 1, hence its cube will end in 1.

(ii) Unit digit of number 358 is 8, hence its cube will end in 2.

(iii) Unit digit of number 419 is 9, hence its cube will end in 9.

(iv) Unit digit of number 725 is 5, hence its cube will end in 5.

(v) Unit digit of number 854 is 4, hence its cube will end in 4.
(vi) Unit digit of number 987 is 7, hence its cube will end in 3.

(vii) Unit digit of number 752 is 2, hence its cube will end in 8.

(viii) Unit digit of numbers 893 is 3, hence its cube will end in 7.

9. Find the cubes of the following numbers:

(i) -13

(ii) $3\frac{1}{5}$

(iii) $-5\frac{1}{7}$

Solution:

(i) Cube of -13 = $(-13)^3$

= (-13) × (-13) × (-13)

= -2197

(ii) Cube of $3\frac{1}{5}$

$= \left(\frac{16}{5}\right)^3 = \frac{(16\times16\times16)}{(5\times5\times 5)}$

$= \frac{4096}{125}$

$= 32\frac{96}{125}$

(iii) Cube of $-5\frac{1}{7}$

$= \left(-\frac{36}{7}\right)^3 = \frac{(-36\times-36\times-36)}{(7\times7\times7)}$

$= -\frac{46656}{49}$

$= -136\frac{8}{343}$

Exercise 4.2

1. Find the cube root of each of the following numbers by prime factorization:

(i) 12167
(ii) 35937
(iii) 42875
(iv) 21952
(v) 373248
(vi) 32768
(vii) 262144
(viii) 157464
Solution:

(i) $\sqrt[3]{12167}$

23	12167
23	529
23	23
	1

$= \sqrt[3]{23 \times 23 \times 23}$

$= (23^3)^{\frac{1}{3}} = 23^{\frac{1}{3} \times 3}$

$= 23^1 = 23$

Thus, the cube root of 12167 is 23.

(ii) $\sqrt[3]{35937}$

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

$= \sqrt[3]{3 \times 3 \times 3 \times 11 \times 11 \times 11}$

$= 3 \times 11 = 33$

Thus, the cube root of 35937 is 33.

(ii) $\sqrt[3]{42875}$

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7}$

$= 3 \times 7 = 35$

Thus, the cube root of 42875 is 35.

(iii) $\sqrt[3]{21952}$

2	21952
2	1076
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$= 2 \times 2 \times 7 = 28$

Thus, the cube root of 21952 is 28.

(v) $\sqrt[3]{373248}$

2	373248
2	186624
2	93312
2	46656
2	23328
2	1164
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$\overline{\times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times}$

$2 \times 3 \times 3 = 72$

ube root of 373248 is 72.

$\overline{58}$

8

4

$\overline{\times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times}$

$2 \times 2 \times 2 = 32$

ube root of 32768 is 32.

14

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 2 \times 2 \times 2 \times 3 \times 3 = 72$

Thus, the cube root of 373248 is 72.

(vi) $\sqrt[3]{32768}$

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= 2 \times 2 \times 2 \times 2 \times 2 = 32$

Thus, the cube root of 32768 is 32.

(vii) $\sqrt[3]{262144}$

2	262144
2	131072
2	65536
2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Thus, the cube root of 262144 is 64.

(viii) $\sqrt[3]{157464}$

2	157464
2	78732
2	39366
3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= 2 \times 3 \times 3 \times 3 = 54$

Thus, the cube root of 157464 is 54.

2. Find the cube root of each of the following cube numbers through estimation.

(i) 19683
(ii) 59319
(iii) 85184
(iv) 148877
Solution:

(i) 19683

Grouping in 3's from right to left, we have 19,683

In the first group, 683 the unit digit is 3

So, the cube root will end with 7 and in the second group, 19

Cubing $2^3 = 8$ and $3^3 = 27$

As, $8 < 19 < 27$

The ten's digit of the cube root will be 2

Thus, the cube root of 19683 is 27.

(ii) 59319

grouping in 3's from right to left, we have 59,319

In first group 319, and unit digit is 9

So, the unit digit of its cube root will be 9 and in the second group, 59

Cubing $3^3 = 27$ and $4^3 = 64$

As, $27 < 59 < 64$

The ten's digit of the cube root will be 3

Thus, the cube root of 59319 is 39.

(iii) 85184

grouping in 3's from right to left, we have 85,184

In the first group 184, the unit digit is 4

So, the unit digit of its cube root will be 4 and in the second group, 85

Cubing $4^3 = 64$ and $5^3 = 125$

As, $64 < 85 < 125$

The ten's digit of cube root will be 4

Thus, the cube root of 85184 is 44.

(iv) 148877

Grouping in 3's, from right to left, we have 148,877

In the first group 877, unit digit is 7

So, the unit digit of cube root will be 3 and in the second group, 148

Cubing $5^3 = 125$, $6^3 = 216$

$125 < 148 < 216$

The ten's digit of cube root will be 5

Thus, the cube root of 148877 is 53.

3. Find the cube root of each of the following numbers:

(i) -250047 **(ii)** $-\frac{64}{1331}$

(iii) $4\frac{17}{27}$ **(iv)** $5\frac{1182}{2197}$

Solution:

(i) $\sqrt[3]{-250047} = -\sqrt[3]{250047}$

3	250047
3	83349
3	27783
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$

$= 3 \times 3 \times 7 = 63$

$= -\sqrt[3]{250047} = -63$

Thus, the cube root of -250047 is -63.

(ii) $\sqrt[3]{\frac{-64}{1331}} = \sqrt[3]{\frac{64}{1331}}$

Performing prime factorization for both the numerator and denominator, we have

2	64
2	32
2	16
2	8
2	4
2	2
	1

11	1331
11	121
11	11
	1

$$= \sqrt[3]{\frac{2\times2\times2\times2\times2\times2}{11\times11\times11}}$$

$$= \frac{\sqrt[3]{2\times2\times2\times2\times2\times2}}{\sqrt[3]{11\times11\times11}}$$

$$= \frac{2\times2}{11} = \frac{4}{11}$$

$$\therefore \sqrt[3]{\frac{-64}{1331}} = \frac{-4}{11}$$

(iii) $\sqrt[3]{4\frac{17}{27}} = \sqrt[3]{\frac{108+17}{27}} = \sqrt[3]{\frac{125}{27}}$

$$= \sqrt[3]{\frac{5\times5\times5}{3\times3\times3}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{3\times3\times3}} = \frac{5}{3}$$

$$\therefore \sqrt[3]{4\frac{17}{27}} = \frac{5}{3}$$

(iv) $5\frac{1182}{2197} = \frac{5\times2197+1182}{2197}$

$$= \frac{10985+1182}{2197} = \frac{12167}{2197}$$

23	12167
23	569
23	23
	1

13	2197
13	169
13	13
	1

$$= \frac{\sqrt{23\times23\times23}}{\sqrt{13\times13\times13}} = \frac{23}{23}$$

$\therefore \sqrt[3]{5\frac{1182}{2197}} = \frac{23}{13}$

4. Evaluate the following:

(i) $\sqrt[3]{512 \times 729}$

(ii) $\sqrt[3]{(-1331) \times 3375}$

Solution:

(i) $\sqrt[3]{512 \times 729}$

$= \sqrt[3]{512} \times \sqrt[3]{729}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} \times \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= (2 \times 2 \times 2) \times (3 \times 3) = 72$

$\therefore \sqrt[3]{512 \times 729} = 72$

(ii) $\sqrt[3]{(-1331) \times (3375)}$

$= \sqrt[3]{-1331} \times \sqrt[3]{3375}$

$= -\sqrt[3]{1331} \times \sqrt[3]{3375}$

3	3375
3	1126
3	375
5	125
5	25
5	5
	1

$= -\sqrt[3]{11 \times 11 \times 11} \times \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$

$= -11 \times 3 \times 5 = -11 \times 15 = -165$

$\therefore \sqrt[3]{(-1331) \times (3375)} = -165$

5. Find the cube root of the following decimal numbers:

(i) 0.003375

(ii) 19.683

Solution:

(i) $\sqrt[3]{0.003375} = \sqrt[3]{\frac{3375}{1000000}}$

$= \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$

10	1000000
10	100000
10	10000
10	1000
10	100
10	10
	1

3	3375
3	1126
3	375
5	125
5	25
5	5
	1

$$= \frac{\sqrt[3]{3\times3\times3\times5\times5\times5}}{\sqrt[3]{10\times10\times10\times10\times10\times10}}$$

$$= \frac{3\times5}{10\times10} = \frac{15}{100} = 0.15$$

(i) $\sqrt[3]{19.683} = \sqrt[3]{\frac{19683}{1000}}$

$$= \frac{\sqrt[3]{19683}}{\sqrt[3]{1000}}$$

3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$= \frac{\sqrt[3]{3\times3\times3\times3\times3\times3\times3\times3\times3}}{\sqrt[3]{10\times10\times10}}$$

$= \frac{3\times3\times3}{10} = \frac{27}{10} = 2.7$

$= 3 \times 3 \times 7 = 63$

$= -\sqrt[3]{250047} = -63$

6. $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

Solution:

$\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

$= \sqrt[3]{3 \times 3 \times 3} + \sqrt[3]{0.2 \times 0.2 \times 0.2} + \sqrt[3]{0.4 \times 0.4 \times 0.4}$

$= 3 + 0.2 + 0.4 = 3.6$

7. Multiply 6561 by the smallest number so that product is a perfect cube. Also, find the cube root of the product.

Solution:

Performing prime factorization of 6561, we get

$6561 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

$6561 = (3 \times 3 \times 3) \times (3 \times 3 \times 3) \times 3 \times 3$

After grouping of the equal factors in 3's, it's seen that 3 × 3 is left ungrouped in 3's.

In order to complete it in triplet, we should multiply it by 3.

Hence, required smallest number = 3

and cube root of the product = 3 × 3 × 3 = 27

8. Divide the number 8748 by the smallest number so that the quotient is a perfect cube. Also, find the cube root of the quotient.

Solution:

Given number is 8748

On prime factorizing, we get

8748 = 2 × 2 × 3 × 3 × 3 × 3 × 3 × 3 × 3

3	8748
3	2916
3	972
3	324
3	108
3	36
3	12
2	4
2	2
	1

Grouping of the equal factor in 3's, it's seen that $2 \times 2 \times 3$ is left without grouping.

$8748 = 2 \times 2 \times 3 \times (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

Hence, on dividing the number 8748 by 12, we get 729

And, the cube root of 729 is $3 \times 3 = 9$.

9. The volume of a cubical box is 21952 m^3. Find the length of the side of the box.

Solution:

Given, the volume of a cubical box is 21952 m^2.

We know that,

Its edge = $\sqrt[3]{21952}$ m

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$ m

$= 2 \times 2 \times 7 = 28$ m

Thus, the length of the side of the box is 28 m.

10. Three numbers are in the ratio 3 : 4 : 5. If their product is 480, find the numbers.

Solution:

Given,

Three numbers are in the ratio 3:4:5 and their product = 480

let's assume the numbers to be 3x, 4x and 5x, then we have

$3x \times 4x \times 5x = 480$

$\Rightarrow 60\ x^3 = 480$

$\Rightarrow x^3 = \frac{480}{60} = 8 = (2)^3$

$\therefore x = 2$

Thus, the number are 2×3, 2×4 and $2 \times 5 = 6$, 8 and 10

11. Two numbers are in the ratio 4: 5. If difference of their cubes is 61, find the numbers.

Solution:

Given,

Two numbers are in the ratio = 4 : 5

Difference between their cubes = 61

Let's assume the numbers to be 4x and 5x

So, we have

$(5x)^3 - (4x)^3 = 61$

$125x^3 - 64x^3 = 61$

$61x^3 = 61$

$\Rightarrow x^3 = 1 = (1)^3$

$\therefore x = 1$

Hence, $4x = 4 \times 1 = 4$ and $5x = 5 \times 1 = 5$

Therefore, the numbers are 4 and 5

12. Difference of two perfect cubes is 387. If the cube root of the greater of two numbers is 8, find the cube root of the smaller number.
Solution:

Given,

The difference in two cubes = 387

And, the cube root of the greater number = 8

So, the greater number = $(8)^3 = 8 \times 8 \times 8 = 512$

Hence, the second number = 512 – 387 = 125

Thus,

The cube root of 125 is

$= \sqrt[3]{125} = \sqrt[3]{5 \times 5 \times 5} = 5$

Mental Maths

Question 1: Fill in the blanks:

(i) Cube of an even natural number is ………….

(ii) Cubes of natural numbers are called ………….

(iii) Cube of a natural number whose unit digit is 7, ends with digit …

(iv) If a natural number has one zero at its end, then its cube will contain ……… zero at the end.

(v) Finding cube root is the inverse operation of finding ………….

(vi) If a number can be expressed as the triplets of equal prime factors, then it is called ………….

Solution:

(i) Cube of an even natural number is even number.

(ii) Cubes of natural numbers are called perfect cube.

(iii) Cube of a natural number whose unit digit is 7, ponds with digit 3.

(iv) If a natural number has one zero at its end, then its cube will contain three zero at the end.

(v) Finding cube root is the inverse operation of finding cube.

(vi) If a number can be expressed as the triplets of equal prime factors, then it is called perfect cube.

Question 2: State whether the following statements are true (T) or false (F):

(i) Cube of any odd number is even.

(ii) A perfect cube does not end with two zeros.

(iii) If square of a number ends with 5, then its cube ends with 25.

(iv) There is no perfect cube which ends with 8.
(v) The cube of a two digit number may be a three digit number.
(vi) The cube of a single digit number may be single digit number.
(vii) If a number is multiplied by 2 then its cube is multiplied by 8.
Solution:
(i) Cube of any odd number is even. False
Correct:
Cube of an odd number is odd.
(ii) A perfect cube does not end with two zeros.
True
(iii) If square of a number ends with 5,
then its cube ends with 25. False
Correct:
Square never ends with only 5, its ends with 25.
(iv) There is no perfect cube which ends with 8.
False
Correct:
Perfect cube can also ends with 8,
for example, the cube of a number whose unit digit is x, will end with 8.
(v) The cube of a two digit number may be a three digit number. False
Correct:
More than three digits.
(vi) The cube of a single digit number may be a single digit number.
True
(vii) If a number is multiplied by 2 then its cube is multiplied by 8. True

Multiple Choice Questions

Choose the correct answer from the given four options (3 to 12):

Question 3: Cube of a negative number is
(a) negative

(b) positive
(c) negative or positive
(d) None of these.
Solution:
The cube of a negative number is negative. (a)

Question 4: The unit digit of cube of 476 is
(a) 4
(b) 6
(c) 8
(d) 2
Solution:
The unit digit of cube of 476 is 6. (b)

Question 5: Cube of (-8) is
(a) -512
(b) 512
(c) -64
(d) 64
Solution:
Cube of (-8) is (-512). (a)

Question 6: Cube of $\left(\frac{-3}{7}\right)$ is

(a) $\frac{27}{343}$ **(b)** $\frac{-27}{343}$

(c) $\frac{9}{49}$ **(d)** $\frac{-9}{49}$

Solution:

Cube of $\left(\frac{-3}{7}\right)$ is $\frac{-27}{343}$. (b)

Question 7: Cube root of -1331 is

(a) 11

(b) 21

(c) -11

(d) -21

Solution:

Cube root of -1331 is -11.

Question 8: Cube root of 2744 is

(a) 16

(b) 18

(c) -14

(d) 14

Solution:

Cube root of 2744 is $2 \times 7 = 14$.

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

Question 9: Cube root of $1\frac{127}{216}$ is

(a) $\frac{6}{7}$ **(b)** $\frac{-7}{6}$

(c) $1\frac{1}{6}$ **(d)** $\frac{-6}{7}$

Solution:

Cube root of $1\frac{127}{216}$

$$= \sqrt[3]{\frac{343}{216}} = \sqrt[3]{\frac{7\times7\times7}{6\times6\times6}} = \frac{7}{6} = 1\frac{1}{6} \text{ (c)}$$

Question 10: The smallest number by which 192 should be multiplied to make it a perfect cube is

(a) 9
(b) 6
(c) 3
(d) 2

Solution:

The smallest number by which 192 should be multiplied to make it a perfect cube is 3 × 3 = 9 (a)

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Question 11: The smallest number by which 686 should be divided to make it a perfect cube is

(a) 1
(b) 2
(c) 3

(d) 4

Solution:

The smallest number by which 686
should be divided to make it a perfect cube is 2. (b)

2	686
7	343
7	49
7	7
	1

Question 12: The volume of a cube is 729 m^3. Length of its side is

(a) 3 m

(b) 6 m

(c) 9 m

(d) 27 m

Solution:

Volume of a cube = 729 m^3

$\therefore$ Side = $\sqrt[3]{729}$

Value Based Question

Question 1. A school decided to award prizes to their students for three values honesty, punctuality and obedience. If the number of students getting prizes for honesty, punctuality and obedience are in the ratio 1: 2: 3 and their product is 162, find the number of students getting prizes for each value. Which quality you prefer to be rewarded most and why? What values are being promoted?

Solution:

Ratio in three types of awards = 1 : 2 : 3
Product = 162
Let awards be x, 2x, 3x
Then $x \times 2x \times 3x = 162 \Rightarrow 6x^3 = 162$
$\Rightarrow x^3 = \frac{162}{6} = 27 = (3)^3$
$\Rightarrow x = 3$
Number of students of first award = $3 \times 1 = 3$
Number of students of second award = $3 \times 2 = 6$
Number of students of third award = $3 \times 3 = 9$
The awards for honesty, punctuality
and obedience promote the students to be a good citizens.

Higher Order Thinking Skills (Hots)

Question 1: Find the volume of a cubical box if the cost of painting its outer surface is ₹1440 at the rate of ₹15 per sq. m.

Solution:
Total cost of painting the surface of a cubical box = ₹1440
and rate = ₹15 per sq. m
Total surface = $\frac{1440}{15} = 96\ m^2$
$\therefore$ Side = $\sqrt{\frac{96}{6}} = \sqrt{16} = 4m$
Now volume = $(Side)^3 = (4)^3 = 64\ m^{32}$

Question 2: In a Maths lab there are some cubes and cuboids of the following measurements:

(i) one cube of side 4 cm

(ii) one cube of side 6 cm

(iii) 3 cuboids each of dimensions 4 cm × 4 cm × 6 cm and 3 cuboids each of the dimensions 4 cm × 6 cm × 6 cm.

A student wants to arrange these cubes and cuboids in the form of a big cube. Is it possible for him/her to arrange them in the form of a big cube? If yes, then find the length of side of new cube so formed.

Solution:

(i) One cube of side = 4 cm

∴ Volume = $(4)^3 = 64$ cm^3

(ii) One cube of side = 6 cm

Volume = $(6)^3 = 216$ cm^3

(iii) 3 cuboids of the size 4 cm × 4 cm × 6 cm

Volume = $4 \times 4 \times 6 \times 3 = 288$ cm^3

and 3 cuboids of size $4 \times 6 \times 6 \times 3$

= 432 cm^3

Total volume = 64 + 216 + 288 + 432

= 1000 cm^3

∴ Volume of big single cube = 1000 cm^3

and side = $\sqrt[3]{1000} = \sqrt[3]{10 \times 10 \times 10} = 10$ cm

Yes, possible and side = 10 cm

Check your progress

Question 1: Show that each of the following numbers is a perfect cube. Also find the number whose cube is the given number:

(i) 74088
(ii) 15625
Solution:

(i) 74088

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7$
Grouping the same kind of prime factors in 3's,
we see that no factor has been left ungrouped.
So, 74088 is a perfect cube and its cube root is $2 \times 3 \times 7 = 42$

(ii) 15625

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$= 5 \times 5 \times 5 \times 5 \times 5 \times 5$

Grouping the same kind of prime factors

we see that no factor is left ungrouped.

So, 15625 is a perfect cube and its cube root is $5 \times 5 = 25$.

Question 2: Find the cube of the following numbers:

(i)-17

(ii) $-3\frac{4}{9} = -\frac{31}{9}$

Solution:

(i) Cube of-17 = (-17) × (-17) × (-17) = -4913

(ii) Cube of $-3\frac{4}{9} = -\frac{31}{9} = -\frac{31}{9} \times -\frac{31}{9} \times -\frac{31}{9} = -\frac{29791}{729}$

$= -40\frac{631}{729}$

Question 3: Find the cube root of each of the following numbers by prime factorization:

(i) 59319

(ii) 21952

Solution:

(i) $\sqrt[3]{59319}$

3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

$= \sqrt[3]{3 \times 3 \times 3 \times 13 \times 13 \times 13}$

$= 3 \times 13 = 39$

(i) $\sqrt[3]{21952}$

2	21952
2	10976
2	5488
2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$= 2 \times 2 \times 7 = 28$

Question 4: Find the cube root of each of the following numbers:
(i) -9261
(ii) $2\frac{43}{343}$

(iii) 0.216

Solution:

(i) $\sqrt[3]{-9261} = -\sqrt[3]{9261}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

(ii) $2\frac{43}{343} = \frac{686+43}{343} = \frac{729}{343}$

$\therefore \sqrt[3]{\frac{729}{343}} = \frac{\sqrt[3]{729}}{\sqrt[3]{343}}$

3	729
3	243
3	81
3	27
3	9
3	3
	1

7	343
7	49
7	7
	1

$= \frac{\sqrt[3]{3\times3\times3\times3\times3\times3}}{\sqrt[3]{7\times7}}$

$= \frac{3\times3}{7} = \frac{9}{7} = 1\frac{2}{7}$

(iii) $0.216 = \frac{216}{1000}$

$\therefore \sqrt[3]{\frac{216}{1000}} = \frac{\sqrt[3]{216}}{\sqrt[3]{1000}}$

2	216
2	108
2	54
3	27
3	9
3	3
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

$= \frac{\sqrt[3]{2\times2\times2\times3\times3\times3}}{\sqrt[3]{2\times2\times2\times5\times5\times5}}$

$= \frac{2\times3}{2\times5} = \frac{6}{10} = 0.6$

Question 5: Find the smallest number by which 5184 should be multiplied so that product is a perfect cube. Also find the cube root of the product.

Solution:

Factorising 5184

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	9
3	3
	1

$\therefore \sqrt[3]{\frac{216}{1000}} = \frac{\sqrt[3]{216}}{\sqrt[3]{1000}}$

2	216		2	1000
2	108		2	500
2	54		2	250
3	27		5	125
3	9		5	25
3	3		5	5
	1			1

$= \frac{\sqrt[3]{2\times2\times2\times3\times3\times3}}{\sqrt[3]{2\times2\times2\times5\times5\times5}}$

$= \frac{2\times3}{2\times5} = \frac{6}{10} = 0.6$

Question 5: Find the smallest number by which 5184 should be multiplied so that product is a perfect cube. Also find the cube root of the product.

Solution:

Factorising 5184

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	9
3	3
	1

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

Grouping the same kind of prime factors is 3's, we see that one factor 3 is left ungroup.

So, to complete it in 3's, we must multiply $3 \times 3 = 9$.

Required least number = 9 and cube root of $5184 \times 9 = 46656$

$= 2 \times 2 \times 3 \times 3 = 36$

Question 6: Find the smallest number by which 8788 should be divided so that quotient is a perfect cube. Also, find the cube root of the quotient.

Solution:

Factorising 8788

2	8788
2	4394
13	2197
13	169
13	13
	1

$= 2 \times 2 \times 13 \times 13 \times 13$

Grouping of the same kind of factors, we see that 2×2 has been left ungrouping.

So, $2 \times 2 = 4$ is the least number to divide it

$\therefore 8788 \div 4 = 2197$ and its cube root = 13

Question 7: Find the side of a cube whose volume is 4096 m^3.

Solution:

Volume of a cube = 4096 m^3

∴ Its side = $\sqrt[3]{4096}$ m

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= 2 \times 2 \times 2 \times 2 = 16\ m$